Target Reliability of Concrete Structures Governed by Serviceability Limit State Design

by

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Abstract

Structures are typically designed for the ultimate limit state (ULS) while the serviceability limit state (SLS) is checked. However, in many cases the design is governed by SLS requirements. The target reliability for the ULS $\beta_{t,ULS}$ is recommended based on cost optimisation and back calibration of existing practice. The current recommendation in ISO2394 is $\beta_{t,SLS} = 1.5$ for the irreversible SLS, although, it is unclear how this value was obtained.

The aim of this research is to determine reasonable $\beta_{t,SLS}$ values for all serviceability requirements with a focus on concrete structures governed by SLS design. The approach taken, by this thesis, is based on reliability-based economic optimisation in a generalised context, taking into account a range of failure consequences and cost classes.

A generalised framework is established to perform a reliability analysis. The limit state function g for the SLS is generalised to $g = 1 - \eta E$. This equation is based on the assumption that the design limit is deterministic and the action-resistance effect E is the only random variable. From the generic limit state equation, a generalised decision parameter d is determined, and the reliability level $\beta(d)$ is determined through a FORM analysis.

Through economic optimisation of the generalised $\beta(d)$ values, a parametric table of $\beta_{t,SLS}$ values is obtained. The table accounts for a range of cost ratios $C_1/C_f = [0.5 - 100]$ (relating the failure costs C_f to the costs per unit of the decision parameter C_1) and coefficient of variation of the action-resistance effect $V_E = [0.05 - 0.50]$. $\beta_{t,SLS}$ is observed to systematically increase as the cost ratio increases and V_E decreases. The current recommendation of 1.5 seems reasonable from a

practical perspective for relatively low cost ratios.

Two examples of the application of $\beta_{t,SLS}$, from the generic development, are provided. $\beta_{t,SLS}$ of a water retaining structure, governed by cracking, and $\beta_{t,SLS}$ of a simply supported beam, for deflections, are investigated. E is the product of either the mean-maximum crack width $w_{m,max}$ or mean-maximum deflections $\delta_{m,max}$ and the model factor θ. V_E is then determined through Monte Carlo analysis and C_f/C_1 is quantified for the two structures.

Once V_E and C_f/C_1 are known, $\beta_{t,SLS}$ is obtained from the generic development. Through economic optimisation of the specific $\beta(d)$ values, $\beta_{t,SLS}$ is obtained for the same costs. These target values are compared to $\beta_{t,SLS}$ from the generic development. The discrepancy identified in the $\beta_{t,SLS}$ values for the examples is due to the inefficiency of the generic decision parameter to increase the reliability level of the structure compared to the efficiency of the specific decision parameters.

Opsomming

Strukture word tipies vir die grenstoestand van swigting (ULS) ontwerp terwyl die grenstoestand van diensbaarheid (SLS) slegs geverifieer word. In baie gevalle is die SLS bepalend vir die ontwerp. Die teiken betroubaarheid vir ULS, $\beta_{t,ULS}$ is gebaseer op koste optimisering en kalibrasie na bestaande praktyk van bestaande praktyk. Die huidige aanbeveling in ISO2394 is $\beta_{t,SLS} = 1.5$ vir die onomkeerbare SLS, alhoewel die oorsprong van hierdie waarde onbekend is.

Die doel van hierdie studie is om redelike $\beta_{t,SLS}$ waardes vir alle diensbaarheid vereistes te bepaal met die klem op betonstrukture wat deur die SLS ontwerp bepaal word. Die benadering, wat deur hierdie studie gevolg word, is gebaseer op ekonomiese optimisering binne 'n algemene konteks met betrekking tot betroubaarheid, met inagneming van 'n verskeidenheid swigting gevolge en kosteklasse.

'n Algemene raamwerk om 'n betroubaarheidsanalise uit te voer word bevestig. Die grenstoestandsfunksie, g, vir die SLS word veralgemeen na $g = 1 - \eta E$. Hierdie vergelyking is gegrond op die aanname dat die limiet deterministies is en die aksie-weerstand-effek, E, is die enigste ewekansige veranderlike. Vanuit die generiese limietstaatvergelyking word 'n algemene besluitparameter d bepaal, en die betroubaarheidsvlak, $\beta(d)$, word bepaal deur middel van 'n EOBM (Eerste Orde Betroubaarheid Metode) analise.

Deur ekonomiese optimisering van die algemene $\beta(d)$ waardes, word 'n parametriese tabel van $\beta_{t,SLS}$ waardes verkry. Die tabel neem 'n verskeidenheid kosteverhoudings $C_1/C_f = [0.5 - 100]$ (Wat die falingskoste C_f tot die koste per eenheid van die besluitparameter C_1 vergelyk) en koffisint van variasie van die aksie-weerstand effek

$V_E = [0.05 - 0.50]$ in ag. Dit word opgemerk dat $\beta_{t,SLS}$ stelselmatig toeneem namate die kosteverhouding toeneem en V_E afneem. Die huidige aanbeveling van 1.5 blyk redelik vanuit 'n praktiese perspektief vir relatief lae koste verhoudings te wees.

Twee voorbeelde van die toepassing van $\beta_{t,SLS}$, vanaf die generiese ontwikkeling, word verskaf. $\beta_{t,SLS}$ van 'n waterhoudende struktuur, beheer deur kraking, en $\beta_{t,SLS}$ van 'n eenvoudig ondersteunde balk, vir defleksie, word ondersoek. E is die produk van die gemiddelde-maksimum kraakwydte $w_{m,max}$ of gemiddelde-maksimum defleksie $\delta_{m,max}$ en die modelfaktor θ. V_E is dan deur middel van 'n Monte Carlo analise bepaal en C_f/C_1 word gekwantifiseer vir die twee tipes strukture.

Sodra V_E en C_f/C_1 bekend is, word $\beta_{t,SLS}$ vanuit die generiese ontwikkeling verkry. Deur ekonomiese optimisering van die spesifieke $\beta(d)$ waardes, word $\beta_{t,SLS}$ verkry vir dieselfde koste. Hierdie teiken waardes word vergelyk met $\beta_{t,SLS}$ van die generiese ontwikkeling. Die teenstrydigheid tussen die $\beta_{t,SLS}$ waardes vir die twee voorbeelde word toegeskryf aan die ondoeltreffendheid van die generiese besluitparameter om die betroubaarheidsvlak van die struktuur te verhoog in vergelyking met die doeltreffendheid van die spesifieke besluitparameters.

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Nomenclature

Abbreviations

APP	Approximation to determine the coefficient of variation
F-LT	Flexure long term
F	Flexure Effect
FORM	First order reliability method
SSB	Simply Supported Beam
T	Tensile Effect
F-ST	Flexure short term
JCSS	Joint Committee of Structural Safety
LN	Lognormal distribution
LT	Long term
MCS	Monte Carlo Simulation
N	Normal distribution
PDF	Probability density function
RT	Risk Tools
SLS	Serviceability limit state
ST	Short term
T-LT	Tension long term
T-ST	Tension short term
ULS	Ultimate limit state
WRS	Water retaining structure

Greek Symbols

$\alpha_{e,LT}$	Modular ratio for the long term concrete modulus
$\alpha_{e,ST}$	Modular ratio for the short term concrete modulus
β	Reliability index
$\beta(A_s)$	Reliability index as a function of A_s
$\beta(h)$	Reliability index as a function of h
$\beta(d)$	Reliability index as a function of d
$\beta(d_{opt})$	Reliability index as a function of the optimum decision parameter d_{opt}, or the target reliability
β_t	Target reliability
$\beta_{t,SLS}$	Target reliability for the serviceability limit state
$\beta_{t,ULS}$	Target reliability for the ultimate limit state
β_ξ	Deflection coefficient taking load duration into account
ϵ_{cm}	Mean strain in the concrete
ϑ	Efficiency factor
ϵ_{sm}	Mean strain in the steel
γ_w	Density of water
μ_i	Mean value of i
μ_E	Mean value of the action-resistance effect
μ_θ	Mean value of the model factor
θ	Model uncertainty
η	Factor for the generalised SLS equation
Φ	Cumulative distribution of the standardized normal distribution

ϕ	The reinforcement diameter
$\rho_{p,eff}$	Ratio of area reinforcement to area of concrete in tension
ρ	Density
σ_s	Stress in tension reinforcement
σ_i	Standard deviation of i
δ_{lim}	Deflection limit
$\delta_{m,max}$	Mean value prediction of the maximum deflection
ξ	Distribution Coefficient
ξ_1	Adjusted ratio of the bond strength

Subscripts

i	Random variable i

Terminology

$\frac{C_f}{C_1}$	Cost ratio
$g(\mathbf{X})$	Limit state function of random variables ($\mathbf{X}$)

Other Symbols

$A_{c,eff}$	Effective area of concrete in tension
A_p	rea of pre- or post tensioned tendons
A_s	Area of tensile reinforcement
A_{s_1}	Area of tensile reinforcement of one bar - effective decision parameter for the WRS
A_{s_L}	Area of tensile reinforcement which results in $E = L$
b	Width
c	Concrete cover

C_0	Initial costs independent of d
C_1	Construction costs dependent on d
C_b	Building or construction costs
C_f	Failure costs
C_m	Expected maintenance costs
C_{tot}	Total costs
D	Diameter of the WRS
d	Decision parameter
d^*	Effective depth to reinforcement
d_{opt}	Optimum decision parameter
E	ULS load effect or SLS action-resistance effect
Z	Safety Margin
E_{cm}	Concrete modulus
$E_{c,eff}$	Effective long term concrete modulus
E_s	Steel modulus
f	Percentage the cracked moment of inertia is of the uncracked moment of inertia
f_{ctm}	Mean value of the concrete tensile strength
f_{cm}	Mean value of the concrete compressive strength
f_{cu}	Concrete compressive strength
f_y	Steel yield strength
H	Height of the WRS
h	Thickness of the WRS or depth of the beam section

$h_{c,eff}$	Effective depth of concrete in tension
h_L	Height of the beam which results in $E = L$
I_1	Moment of inertia for uncracked section
I_2	Moment of inertia for fully cracked section
k_1	Factor taking bond properties into account
k_2	Factor taking the distribution of strain into account
k_3	Factor
k_4	Factor
k_t	Load duration factor
L	SLS limiting design value
L	Span of the beam
G	Permanent load
Q	Variable load
M_s	Flexural force from applied loading
M_{cr}	Cracking moment
p_f	Probability of failure
$p_f(A_s)$	Probability of failure as a function of A_s
$p_f(h)$	Probability of failure as a function of h
$p_f(d)$	Probability of failure as a function of the decision parameter
$p_{f,t}$	Target probability of failure
q	Distributed Load
R	ULS resistance

$s_{rm,max}$	Mean-maximum crack spacing
T_s	Tensile force from applied loading
V_i	Coefficient of variation of i
V_E	Coefficient of variation of the action-resistance effect
V_θ	Coefficient of Variation of the model factor
w_{lim}	Crack width limit
$w_{m,max}$	Mean value prediction of the maximum crack width
w_s	Weight of one bar of reinforcement
x	Depth to the neutral axis
Y	The calculated action-resistance effect
z	Moment lever arm

1. Introduction

The aim of this research project is to develop suitable target reliability values for concrete structures for the serviceability limit state $\beta_{t,SLS}$.

This chapter includes the problem statement along with a motivation as to why the investigation of $\beta_{t,SLS}$ is necessary, especially for structures governed by SLS design. The research goals and objectives of the project are presented, along with an outline of how each objective is to be achieved. Lastly, the layout and structure of the thesis is established.

1.1 Problem statement

In the limit state design approach, the ultimate limit state (ULS) typically governs the design of structures, while the serviceability limit state (SLS) is verified. However, certain structures are governed by SLS requirements; such as maximum crack widths for water retaining structures or stress limits for post-tensioned bridges. The design of concrete bridges, which was historically governed by ULS, is now being governed by serviceability cracking based on the requirements in the new design codes. Due to serviceability requirements governing the design of certain concrete structures, an investigation into the reliability specifications for the SLS is increasing in importance.

Prescribed target reliability values for the ULS $\beta_{t,ULS}$ and the SLS $\beta_{t,SLS}$ are recommended in international and national design standards. The current recommendations in ISO2394 for $\beta_{t,ULS}$ were determined based on cost optimisation and back calibration of existing practice [23]. These values are differentiated based on the estimated cost of increasing safety and the severity of consequences of ULS failure. $\beta_{t,SLS}$ is recommended by ISO2394 as 1.5 for the irreversible SLS, corresponding to a low consequence of failure. It is unclear how $\beta_{t,SLS}$ was determined, it should, however, be determined based on the same principles as for the ULS.

The question arises as to whether or not the current recommendation for $\beta_{t,SLS}$ is suitable, specifically when SLS governs the design.

1.2 Research Goal and Objectives

The goal of this project is to research and develop suitable target reliability values $\beta_{t,SLS}$ for both new and existing concrete structures, suitable for all serviceability conditions, with a focus on structures governed by SLS design.

The key objectives identified to achieve the goal are summarized below:

1. The first objective is to provide background information for the key topics of structural reliability. These topics are limit state design (both ULS and SLS), model uncertainty and target reliability.

2. The second objective is to determine $\beta_{t,SLS}$ for a generic structure through economic optimisation. To calculate $\beta_{t,SLS}$, a generic framework, suitable for the SLS of all concrete structures as well as a range of consequence and cost classes, is established. This is done through the use of a parametric table, which considers the variability of different structures V_E (the coefficient of variation of the action-resistance effect E) with different cost classes C_f/C_1 (the failure costs C_f relating to the costs per unit of the decision parameter C_1). The following sub-objectives are used to develop and implement the generic framework:

   (a) A background into the concept of reliability-based cost optimisation, used to determine the target reliability of a structure, is provided. Three components necessary for a cost optimisation are identified: the decision parameter d, the level of reliability $\beta(d)$, and the costs (construction costs, costs of providing safety, and failure costs).

   (b) A necessary component of cost optimisation for the SLS, $\beta(d)$ is determined. Based on the background into reliability provided by objective 1, a FORM analysis is identified as an appropriate method to determine $\beta(d)$. The relationship between the SLS equation and d is established so $\beta(d)$ can be determined.

   (c) $\beta_{t,SLS}$ is determined through economic optimisation of the generalised variables set out in (a) and (b).

3. The third objective is to show how the generic development of objective 2 may be applied to determine $\beta_{t,SLS}$ for specific structures. A water retaining structure (WRS) is chosen as an extensive example of how to determine $\beta_{t,SLS}$ from the generic development, as its design is governed by the limitations on the crack width. The following sub-objectives illustrate how $\beta_{t,SLS}$ is obtained from the generic development:

   (a) The action-resistance effect E of the WRS is to be identified. E is the predicted crack width which is adjusted to account for model uncertainty. The crack width is based on the random variables of the WRS. As the South African code for WRS is currently being developed based on the Eurocode, the provisions of the Eurocode are used. The crack width varies with the amount of reinforcement.

   (b) The coefficient of variation V_E is estimated. Either Monte Carlo Simulation or a suitable approximation may be used to estimate V_E and the probability density function of E.

   (c) The costs are quantified as the cost of providing reinforcement C_1 and the cost of repairing cracks C_f. From these costs the cost ratio C_f/C_1 can be calculated.

   (d) The values of V_E and C_f/C_1, which are quantified for the WRS, are used to obtain $\beta_{t,SLS}$ from the generic development.

   (e) $\beta_{t,SLS}$ for the WRS is established through economic optimisation. The costs and E of the WRS are used to determine $\beta_{t,SLS}$. A comparison of the results of $\beta_{t,SLS}$, obtained from the generic basis, versus calculating $\beta_{t,SLS}$, through cost optimisation of the specific example, is provided.

4. The fourth objective is to provide an additional example of a different serviceability condition. In this additional example, the deflection of a simply supported beam (SSB) is chosen. The SSB and the WRS share similar sub-objetives, with the differences being:

   (a) The first difference is for 3(a), where E of the SSB is the predicted deflection of the beam adjusted to account for model uncertainty. The deflection of the beam varies with the beam's height.

(b) The second difference is for 3(c), where the costs are quantified as the cost to provide concrete C_1 and the cost to strengthen the beam C_f.

1.3 Thesis organisation

Figure 1.1 provides an outline of the objectives discussed in section 1.2.

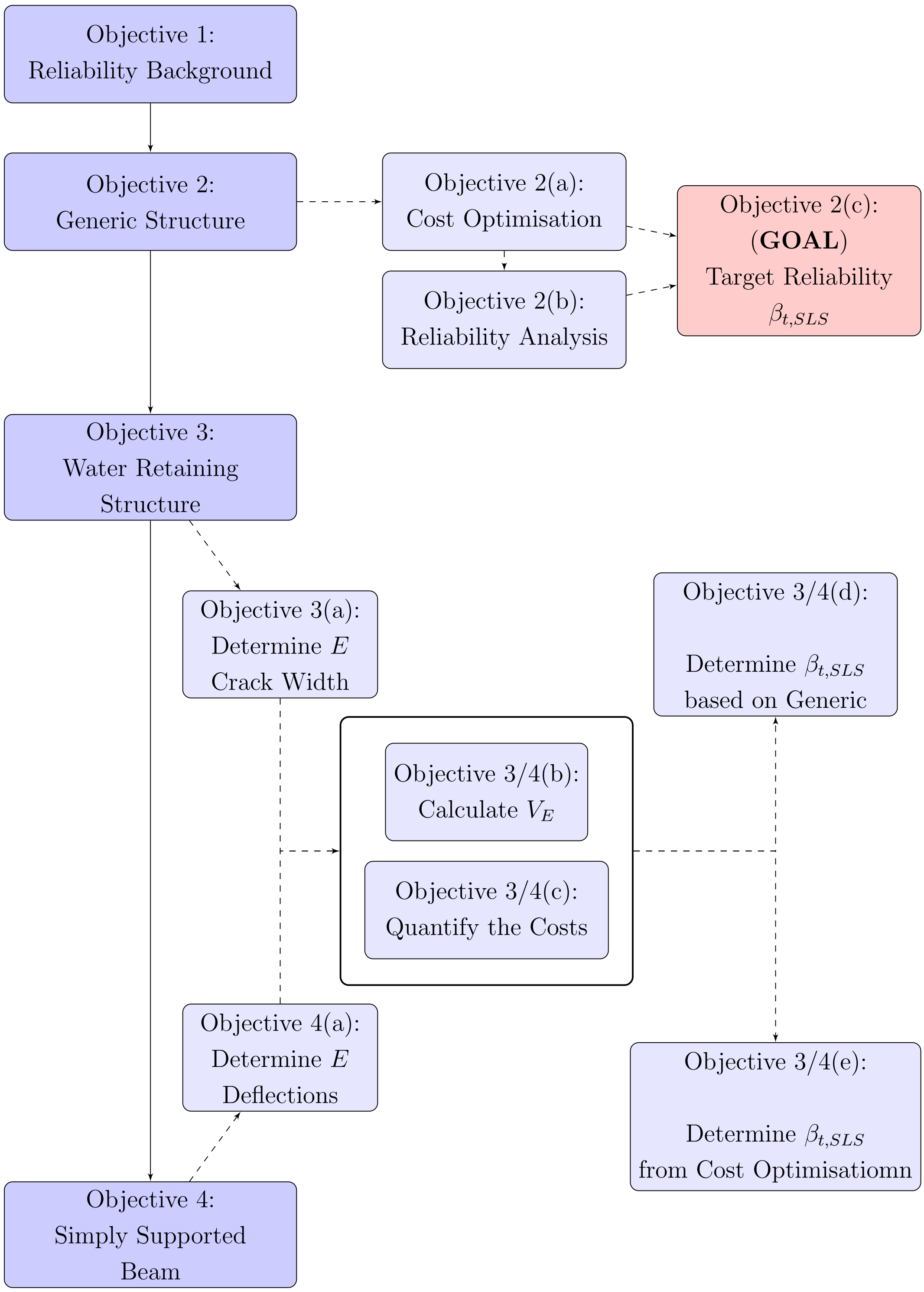


Figure 1.1: Thesis Outline

2. Reliability Background

It is necessary to provide an overview of the basic principles of the reliability theory. First, the limit state design is discussed. Next, the definition of model uncertainty is provided along with the current recommendations. Then, the definitions of reliability and target reliability are provided, with the current recommendations for its respective values. Finally, the first order reliability method (FORM) is described as a suitable method to assess the reliability for the examples in this work.

2.1 Limit States

One of the principles of design is the definition of structural failure. This is described by the two limit states, or the conditions beyond which the structure no longer satisfies its performance criteria [12]. The two limit states, ultimate and serviceability, are associated with different performance requirements of the structure.

The ultimate limit state (ULS) is concerned with maximum load capacity and includes all situations which compromise human and structural safety [12, 25]. The limit state equation for the ULS is shown in equation 2.1 [3, 6, 13]. Structural failure occurs when the realisation of the load E is greater than the actual structural resistance R. The variables of the limit state equation take model uncertainty into account. The model uncertainty of a structure is introduced in section 2.2, but is included in the random variables R and E.

$$g(R, E) = R - E = 0 \tag{2.1}$$

with $g(R, E)$ The limit state g as a function of R and E
R Resistance
E Load effect

The serviceability limit state (SLS) is concerned with the functionality of the structure and includes the conditions for normal use, comfort of people, and appearance of the structure [7, 23]. The SLS is affected by deflections, cracks, stresses, or vibrations caused by the applied loading. The limit state equation for the SLS as defined in EN1990 [7] is shown in equation 2.2. SLS failure occurs when the conditions for normal use are no longer satisfied. In other words, when the action-resistance effect E exceeds the limiting design value L for the SLS.

$$g(L, E) = L - E = 0 \tag{2.2}$$

with	$g(L, E)$	The limit state g as a function of L and E
	L	Limiting design value
	E	Action-resistance effect

The SLS can be either reversible or irreversible [25]. An irreversible SLS occurs if an applied action causes permanent damage to the structure and the structure is considered unfit for use. The SLS is reversible if the structure is unfit for use only while the action is applied and returns to its original state once the action is removed.

2.2 Model Uncertainty

Model uncertainty can be defined as the basic variable related to the accuracy of the physical or statistical models used in design calculations [24, 25]. The model uncertainty θ is a random variable [20], which can be considered independent if it is not related to the variations of the other basic variables.

The uncertainty is due to the mathematical simplifications of physical and probabilistic models [35] or a simplified relationship between the physical behaviour and the basic variables of the model [36]. The model uncertainty takes into account the uncertainty associated with the idealized mathematical descriptions used to model physical behaviour [10]. Due to lack of knowledge or deliberate simplifications of the model, it is accepted that the model is incomplete and inexact [20]. The bias and degree of uncertainty of the model is reflected in the mean value μ_θ and the coefficient of variation V_θ respectively.

For the ULS, model uncertainty can be related to load effects or resistance models individually [20]. However for the SLS, the model uncertainty of the load effect and the resistance is accounted for by one random variable. This is clear from equation 2.2 where L is a prescribed limiting design value.

2.2.1 Procedure for Calculating the Model Factor

Most of the available and widely used model factors are based on intuitive judgement, however, the model factors are currently being developed using experimental data. Holicky et al [20] provide an overview of the methodology required to determine an appropriate model factor based on experimental data.

The steps for determining the model uncertainty are [20]:

1. The first step is to characterize the type of assessment, which determines the scope of work. The model uncertainty is identified based on the importance of the model and is categorized as minor, significant or dominating effect. This characterizes the amount of effort required in determining the model uncertainty.

2. The second step is to determine the dataset, which is made up of the test results. The main attributes of the dataset include: the number of tests, the sample space, the quality of the results, the measured values of the variables, the testing equipment and proper calibration, the boundary conditions, and information regarding the tolerance of the results.

3. The third step is to make observations on the model uncertainty to compile the database. In order to compile the database the following steps are taken: measured material strengths should be used rather than the characteristic values to exclude any design bias and effort should be taken to to obtain the measurements for all the design values. If the material strengths cannot be measured, the mean values can then be used. This does, however, add additional uncertainty (associated with the material strengths) to the model.

4. Finally a statistical assessment of the dataset is performed to determine θ. For unbiased sampling constraints of the basic variables should be included in defining the unbiased design sample space.

ISO2394 - 2015 [24] derives the unknown coefficient θ from the set of observations. This model factor is calculated in equation 2.3.

$$\theta = \frac{y_i}{g(x_i, w_i)} \tag{2.3}$$

with	y_i	Measured (experimental) values
	$g()$	The model
	x_i	Random variables that have been measured from experiments
	w_i	Deterministic variables

2.2.2 Available Recommendations for the Model Factor

The basic variables of model uncertainty for the load effect, resistance and various SLS criteria are presented in table 2.1. The model uncertainty is assumed to be unbiased ($\mu_\theta = 1$) in most cases, although recent studies of the model uncertainty tend to disagree with this assumption, generally with $\mu_\theta > 1$.

For resistance a conservative bias is defined as $\mu_\theta > 1$ [20]. This means that the actual measured structural resistance is more than the predicted resistance. An unconservative bias for resistance is $\mu_\theta < 1$. This is what is expected, as it is not ideal to have a model which over predicts the actual structural resistance. Conversely for cracking the predicted crack width should be less than the measured crack width. Therefore, a conservative bias is defined as $\mu_\theta < 1$ for the crack model. This corresponds with a conservative bias of a loading model.

The values in table 2.1 are the current recommendations from literature that have been investigated or are currently under investigation. The model factor either has a normal (N) or log normal (LN) probability density function (PDF). Holicky [18] recommends values for the model factor on the basis of previous editions of the JCSS Model Code. These values are only indicative values and require further investigations.

Table 2.1: Model Uncertainty θ from Literature

	PDF	**Mean,** μ_θ	V_θ	**Ref**	**Notes**
General Resistance	LN	1	0.1 - 0.3	[35]	
	LN	1	0.1 - 0.3	[21]	
	N	1	0.05 - 0.2	[18]	
	LN	1 - 1.25	0.05 - 0.2	[35]	
Concrete Resistance Flexure	LN	1.2	0.15	[18]	
Load Effect	N	1	0.05 -0.1	[18, 35]	
Cracking	LN	1	0.1 - 0.3	[35]	
	LN	1.05	0.298	[37]	Test Data
	LN	1	0.2 - 0.4	[30]	
	-	1.34	0.42	[4]	Model Code (a)
	-	2.15	0.38	[4]	Model Code (b)
	-	1.09	0.35	[4]	Numerical Simulations (a)
	-	1.53	0.36	[4]	Numerical Simulations (b)
	LN	1	0.3	[18]	
Deflection	LN	1	0.1	[18, 21]	
		0.97	0.06	[11]	Long-term deflections
Stresses	LN	1	0.05	[18]	

McLeod [35] based the model uncertainty of the crack width on what was available and investigated the importance of the model factor on the reliability of the crack model. It was concluded that V_θ had a small influence on the amount of reinforcement required to achieve the target reliability. However this influence, although small is not insignificant and the model factor should not be neglected.

Quan and Gengwei [37] calculated the reliability index based on the maximum crack width of a beam. A model factor from a sample size of 116 was determined. The model factor is calculated as $\theta = 1.5\ w_0/w_{max}$, with w_0 as the observed maximum crack width and w_{max} as the maximum crack width calculated by the model. The coefficient of 1.5 is the coefficient for the long term-effect. From the results, Quan and Gengwei [37] determined a model factor for the long term maximum crack width as $\mu_\theta = 1.05$ and $V_\theta = 0.298$. This is the model factor based on the crack width model provided in the China National Standards.

Markova and Sykora [30] investigated the influence of the coefficient of variation on the reliability index β of a cracking model and expect that a possible bias in the model uncertainty will influence the β values significantly. The reliability of the structure is inversely proportional to coefficient of variation in the model V_θ. Therefore, the larger the uncertainty the lower the reliability and vice-versa. The influence for the range of $V_\theta = [0.2 - 0.4]$ by varying the concrete cover and the reinforcement diameter was investigated. For both the cover and the reinforcement diameter, the target reliability decreases as the cover or reinforcement increases.

From table 2.1, it can be seen that Cervenka et al [4] give uncertainties based on the Model Code and numerical simulations respectively. The first model uncertainty (a) is for the uncertainty associated mean crack width prediction compared to the mean value of the measured crack widths. The second (b) is for the uncertainty associated maximum crack width prediction compared to the maximum measured crack width. However, it states that the numerical simulations are not based on probabilistic models and are, therefore, considered to be subjective. The model uncertainty for the mean crack width represents the model better, as the maximum crack will only occur in one place and not over the entire structure.

For deflections, the model factor is recommended as $\mu_\theta = 1$ and $V_\theta = 0.1$ by Holicky [18] and Honfi et al [21]. Gilbert [11] determined the mean value and coefficient of variation to be $\mu_\theta = 0.97$ and $V_\theta = 0.06$ respectively. This model factor is for long term deflections and takes the effects of creep and shrinkage into account.

2.3 Reliability

Structural reliability can be defined as the ability of a structure to fulfil the specified requirements throughout its service life [7]. The four elements of structural reliability are: a definition of structural failure, an assessment of the service life, an assessment of the probability of failure, and the conditions of structural use [13]. The probability of failure is expressed through the limit state function $g(\mathbf{X})$ such that structural failure occurs if $g(\mathbf{X}) \leq 0$ [7]. The definition of structural failure, or given requirements of the structure, is given by the ULS and SLS [6, 25].

The reliability of a structure is typically expressed in probabilistic terms and includes the safety, serviceability, and durability of the structure. The most recent design method taking structural reliability into account is the probabilistic method [7]. It states that for the design life of the structure the probability of failure should not exceed the design probability of failure [13]. Equations 2.4 and 2.5 show that the probabilistic method is based on the maximum permissible probability of failure $p_{f,t}$ or the corresponding minimum target reliability β_t [13, 24]. The reliability level of a structure can be determined by an assessment of the probability of failure p_f for the reference period. The p_f is related to the reliability index β through equation 2.6 [7].

$$p_f < p_{f,t} \tag{2.4}$$

$$\beta > \beta_t \tag{2.5}$$

$$p_f = \Phi(-\beta) \tag{2.6}$$

with p_f Probability of failure
$p_{f,t}$ Target probability of failure
β Reliability index
β_t Target reliability index
Φ Cumulative distribution function of the standardized normal distribution

It is important to note the difference between the reliability index β and the target reliability index β_t. β refers to the reliability index of a specific structure or the level of reliability of the specific structure, while β_t refers to the target reliability a structure must obtain to be considered reliable. β_t can further be defined by the two limits states as $\beta_{t,ULS}$ and $\beta_{t,SLS}$ for the ULS and SLS respectively.

2.3.1 Recommended β_t Values

EN1990 provides $\beta_{t,ULS}$ values based on the consequence class of the structure. In comparison, ISO2394 and JCSS recommend $\beta_{t,ULS}$ values based on the consequences of failure and the relative costs of safety measures.

Table 2.2 provides the recommended $\beta_{t,ULS}$ for EN1990 [7] for a one and 50 year reference period. The consequence classes, CC, refer to the consequences for human life or the economic, social or environmental consequences. The three consequence classes, CC3 CC2 and CC1, refer to high, medium or low consequences respectively. Bridges typically fall under CC3, while public buildings fall under CC2 structures.

Table 2.3 provides the recommended β_t values for the service life of a structure according to ISO2394 [23]. ISO2394 considers both ULS and SLS in one table, whereas JCSS considers them separately. Another difference is that JCSS considers a one year reference period whereas ISO2394 considers the service life of the structure. A structure's service life is the time period for which the structure does not exceed the minimum requirements of durability and is typically considered to be 50 years for buildings and 100 years for bridges. The $\beta_{t,ULS}$ values according to JCSS [25] are provided in table 2.4.

Table 2.2: $\beta_{t,ULS}$ Values According to EN1990 [7]

Consequence Class	$\beta_{t,ULS}$	
	1 year	50 years
CC3	5.2	4.3
CC2	4.7	3.8
CC1	4.2	3.3

Table 2.3: β_t Values According to ISO2394 [23] for the Service Life

Relative Costs of Safety Measure	Consequences of Failure			
	Small	**Some**	**Moderate**	**Great**
High	0	**1.5**	2.3	3.1
Moderate	1.3	2.3	3.1	3.8
Low	2.3	3.1	3.8	4.3

Table 2.4: $\beta_{t,ULS}$ Values According to JCSS [25] for a One Year Reference Period

Relative Costs of Safety Measure	Minor Consequences of Failure	Moderate Consequences of Failure	Large Consequences of Failure
High	3.1	3.3	3.7
Moderate	3.7	4.2	4.4
Low	4.2	4.4	4.7

For an irreversible SLS a $\beta_{t,SLS} = 1.5$ and for the reversible SLS a $\beta_{t,SLS} = 0$ is accepted for the service life or a 50 year reference period [7, 23]. According to the target $\beta_{t,SLS}$ values of ISO2394 (1998) [23], this corresponds to a *high* relative cost of safety measure and *small/some* consequences of failure (table 2.3). EN1990 [7] recommends for a one year reference period a $\beta_{t,SLS} = 2.9$ which corresponds to the same level of reliability as 1.5 for a 50 year reference period. JCSS [25] suggests $\beta_{t,SLS}$ values for a one year reference period for different relative costs of safety measures. Table 2.5 provides the values recommended in JCSS for irreversible SLS.

Table 2.5: $\beta_{t,SLS}$ Values According to JCSS [25] for a One Year Reference Period

Relative Cost of Safety Measure	$\beta_{t,SLS}$
High	1.3
Normal	1.7
Low	2.3

JCSS, ISO2394 and EN1990 provide recommendations for $\beta_{t,ULS}$ and $\beta_{t,SLS}$ based on different reference periods. ISO2394 [23] recommends β_t based on the expected service life of the structure whereas JCSS [25] recommends β_t for a 1 year and EN1990 [7] for a 1 and 50 year reference period. EN1990 provides a relationship between a 1 year reference period and an n year reference period, as shown in equation 2.7. Emphasis must be placed on the fact that the reliability index for a reference period of one year $\beta_{t,1}$ has the same level of reliability as the corresponding target reliability index $\beta_{t,n}$ [18].

$$\beta_{t,n} = \Phi^{-1}([\Phi(\beta_{t,1})]^n) \tag{2.7}$$

with $\beta_{t,n}$ Target reliability related to a n year reference period
$\beta_{t,1}$ Target reliability related to a 1 year reference period
Φ Cumulative distribution function of the standardized normal distribution
n Reference period

2.3.2 First Order Reliability Method, FORM

The First Order Reliability Method (FORM) is one of techniques available for the reliability analysis of a structure. The FORM algorithm is discussed for the ULS in this section, but the principle does not differ for the SLS.

Structural failure is defined as the inability of the structure or structural element to satisfy the limit state ($g(\mathbf{X}) \leq 0$). Structural reliability is based on the relationship between the random variables ($\mathbf{X}$) of the limit state function.

As an example, R and E are assumed to be two normally distributed variables as shown in figure 2.1 [13]. The design point P is defined at $E = R$. The safety margin Z is the difference between the resistance and the load effect (equation 2.10). β is the number of standard deviations σ_Z (equation 2.11) the safety margin Z is from zero. The corresponding probability of failure is the probability that E is greater than R (equation 2.8).

$$p_f = P(E > R) = P(0 > Z) \tag{2.8}$$

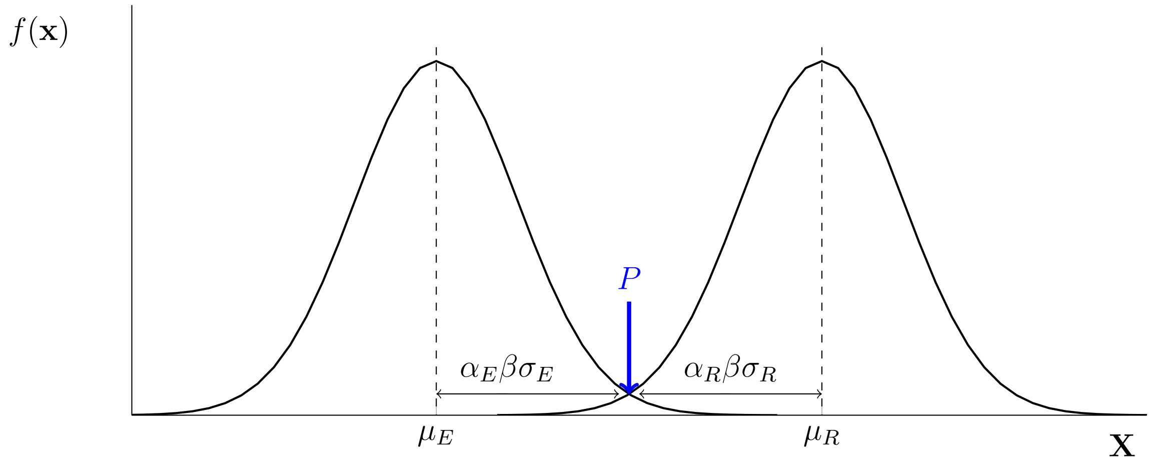


Figure 2.1: Normal distribution of E and R

To determine p_f, a reliability analysis of the structure must be performed. For the two uncorrelated normally distributed variables (shown in figure 2.1), β is found using equation 2.9. The p_f is then determined based on the relationship shown in equation 2.6.

$$\beta = \frac{Z}{\sigma_Z} \tag{2.9}$$

where:

$$Z = R - E \tag{2.10}$$

$$\sigma_Z = \sqrt{\sigma_R^2 + \sigma_E^2} \tag{2.11}$$

with β Reliability index
Z Safety margin
σ_Z Standard deviation of the safety margin
R Resistance
E Load effect
σ_R Standard deviation of the resistance
σ_E Standard deviation of the load effect

If the random variables are not normally distributed and/or the limit state is made up of more than two variables, it is significantly more difficult to evaluate β in such a simple form. FORM is considered to be one of the simplest and more efficient reliability methods [5, 18]. In Handbook 2: *Reliability Backgrounds* [13] FORM is defined as:

"The approximate method of a given iterative algorithm that allows the reliability index to be obtained by using a linear approximation to the limit state surface at the point of minimum distance to the mean point of the variables."

Or more simply FORM gives a linear algorithm to determine the value of β which is the number of standard deviations the design point P is away from the mean. The design point is where the most probable failure point or the line where the limit state equation is $g(\mathbf{X}) = 0$ [5, 13] and is shown in figures 2.1 and 2.2.

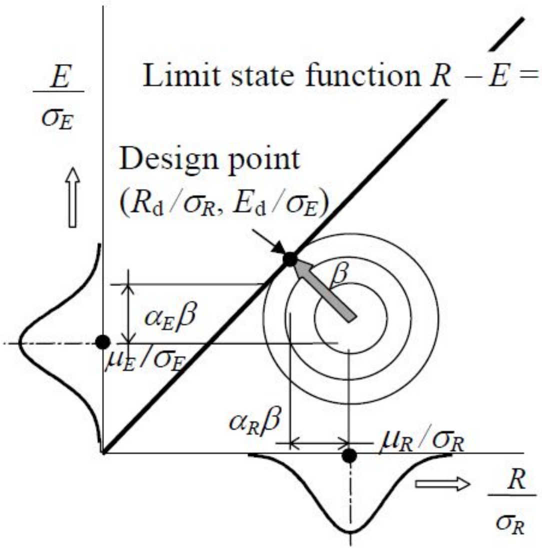


Figure 2.2: Design Point [7]

The limit state function or failure boundary is approximated by a tangent plane in FORM [5]. Figure 2.2 shows this tangent plane along with the design point of the limit state function of two random variables with transformed normal distributions in a two-dimensional diagram. The sensitivity factors (α_R and α_E) are the direction cosines of the normal failure boundary and are considered to be importance measures of R and E in the FORM analysis [18]. The design values (coordinates of the design point) are then determined using equations 2.12 and 2.13.

$$R_d = \mu_R - \alpha_R \beta \sigma_R \tag{2.12}$$

$$E_d = \mu_E - \alpha_E \beta \sigma_E \tag{2.13}$$

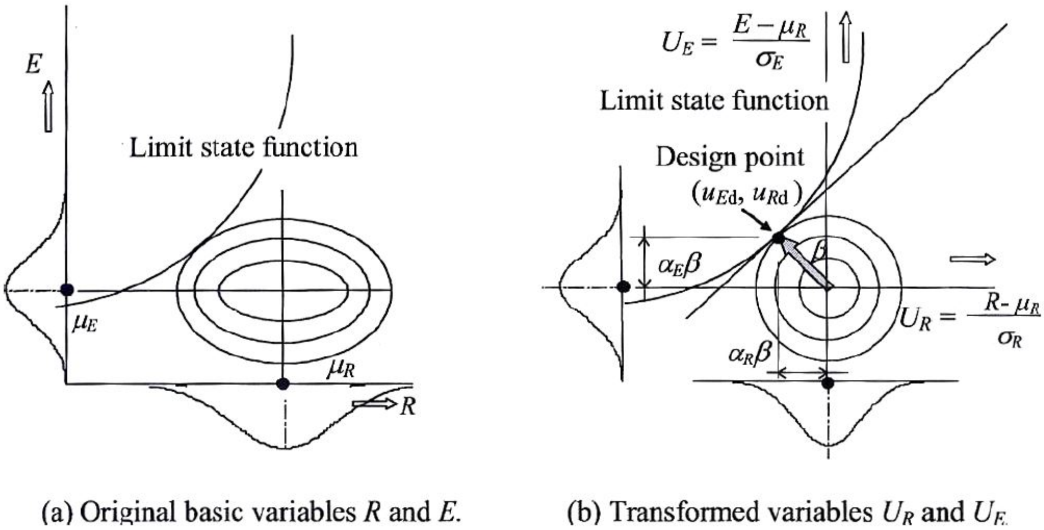


Figure 2.3: FORM [18]

The main steps of FORM summarized in Holicky 2009 [18] are:

1. The basic variables, **X**, are transformed into standardized normal variables, **U**. This is shown in figure 2.3.

2. If the failure boundary is not linear, the failure surface is approximated for a given point.

3. Iteration is done until the design point is found.

4. β is determined, this is the number of standard deviations the mean value is from the design point.

2.4 Concluding Remarks

This chapter summarises the key topics required to perform a reliability analysis. The definition of the limit states is provided, which is later identified to be necessary in performing a reliability analysis. The model uncertainty was defined and the available information regarding model factors related to typical SLS action-resistance effects (for example crack widths or deflections) is provided. A definition of structural reliability and the currently recommended values for β_t are provided. Finally, the FORM algorithm (identified as a suitable method for a reliability analysis) is explained.

3. Generic Structure

To determine a framework, for the target reliability $\beta_{t,SLS}$ for all structures governed by SLS design, a generic structure needs to be considered. $\beta_{t,SLS}$ is established for a range of structures and different cost classes.

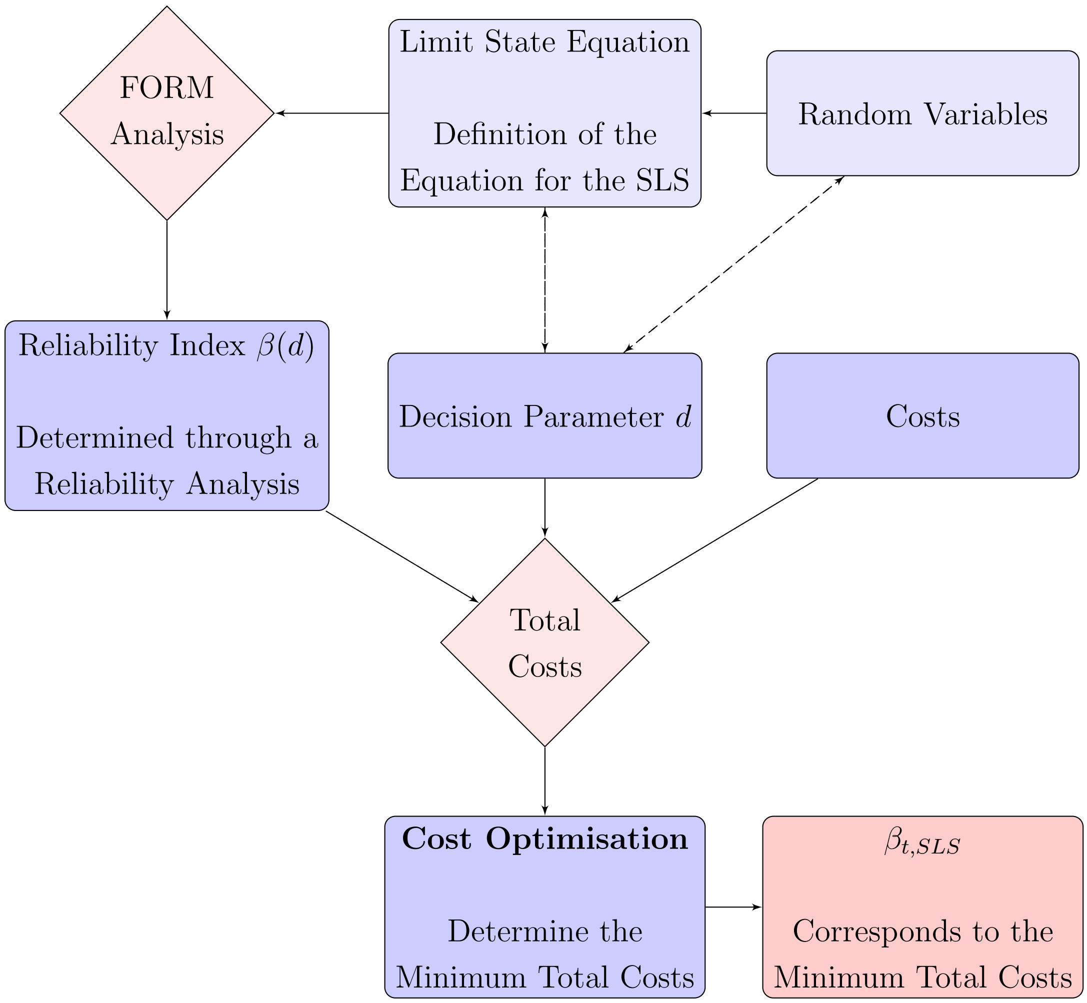


Figure 3.1: Generic Framework

Figure 3.1 outlines the framework used to determine the generic $\beta_{t,SLS}$, which is determined through cost optimisation. First a background into cost optimisation is provided and the components necessary to perform a reliability-based cost optimisation are identified. The three components are: the costs, the decision parameter d and the reliability index $\beta(d)$ which is a function of d. $\beta(d)$ is related to d through the limit state equation and the random variables of the limit state equation. Once all the components are identified, cost optimisation is performed to determine $\beta_{t,SLS}$.

3.1 Cost Optimisation

In order for the structure to be viable from an economic point of view, a balance between the consequences of failure and the costs of safety measures must be achieved. Equation 3.1 [23] contains the total costs. The costs of safety measures include costs which improve the structural reliability. In equation 3.1, the costs of safety measures are defined as the building costs C_b and the maintenance costs C_m The failure consequences include all costs relating to direct and indirect consequences [19, 46].

$$C_{tot} = C_b + C_m + \sum p_f \times C_f \tag{3.1}$$

with	C_{tot}	The total costs
	C_b	The building or construction costs
	C_m	The expected maintenance costs
	C_f	The failure costs
	p_f	Probability of failure

This balance is optimal when the total costs are at a minimum. The fundamental principle of probabilistic optimization is to find the reliability level β_t that would minimize the total costs of the structure over its lifetime. To minimize the total costs of the structure, the cost function needs to take into account some decision parameter d. This is typically a vector of multiple decision parameters. Some of the costs in equation 3.1 depend on d, which leads to equation 3.2. If d is a vector of decision parameters, $\sum C_f p_f(d)$ is the sum of all the expected costs of failures associated with the different decision parameter.

The total costs of a structure C_{tot} as a function of the decision parameter is defined in equation 3.2.

$$C_{tot} = C_0 + C_1 d + \sum C_f p_f(d) \tag{3.2}$$

with d — Decision parameter(s)
C_0 — Initial costs independent of d
C_1 — Costs of providing safety; dependent on d
C_f — Failure costs
$p_f(d)$ — Probability of failure as a function of d
$C_0 + C_1 d$ — Construction costs
$C_f p_f(d)$ — Expected failure costs

The decision parameter(s) d influence(s), for example, the resistance, *serviceability,* durability, maintenance, inspection, or upgrade strategy [48]. Examples of d include: shear or flexural resistances, girder stiffness to control deflections, thickness of components, material strengths, area of reinforcement, and cross sectional properties [19, 43, 46]. For economic optimisation of existing structures, d is typically considered to be the structural resistance concerned with the ULS [44]. The reliability index $\beta(d)$, or its inverse $p_f(d)$ (equation 2.6), is the level of reliability as a function of d which influences the expected failure costs of the structure.

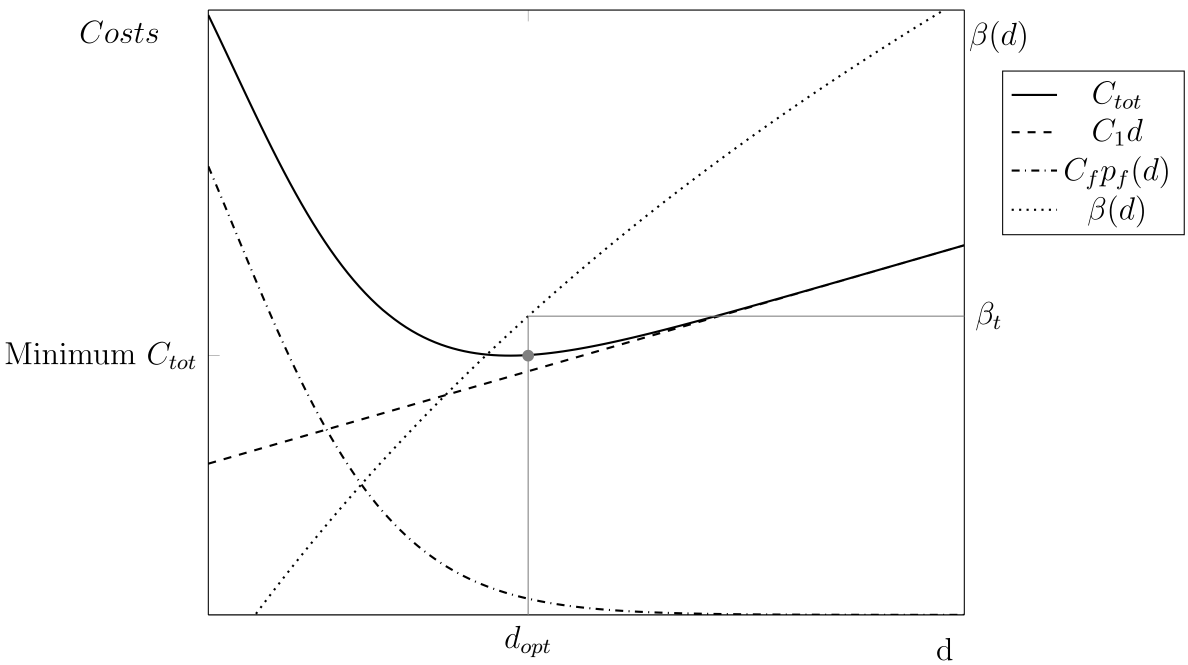


Figure 3.2: Concept of Cost Optimisation

The initial costs C_0 are independent of d. As a simplification the costs C_1 are assumed to have a linear relationship with d and are the costs of providing safety. The combination $C_0 + C_1 d$ is referred to as the construction costs and in general the initial costs are significantly greater than the costs to increase safety ($C_0 \gg C_1 \times d$) [38]. The failure costs C_f correspond to both direct and indirect failure costs and include costs of demolition and reconstruction (ULS failure) or repair (SLS failure), failure costs related to economic consequences, and costs related to societal or environmental consequences [19].

As d increases, $\beta(d)$ increases and C_f subsequently decreases until the point of negligible influence on C_{tot}, as seen in figure 3.2. On the other hand as d increases the influence of the increasing costs $C_1 d$ becomes significant and C_{tot} begins to increase. This point is where the total costs are a minimum and the corresponding d is the optimum decision parameter d_{opt}. The $\beta(d_{opt})$ is known as the target reliability β_t [18, 49].

In order to determine d_{opt} the costs, d and $\beta(d)$ need to be defined. $\beta(d)$ is determined through a reliability analysis and d is determined based on its relationship with the random variables of the limit state equation. The costs are defined through cost ratios discussed in section 3.1.2.

3.1.1 The Costs

From equation 3.2, the total costs of the structure are split into three categories based on their relationship with the decision parameter [2]. This section looks at the different costs and how they relate to each other (cost ratios).

The initial costs C_0 do not have an influence on the β_t. This is due to the fact that C_0 only serves to increase or decrease C_{tot} and does not shift the position of the minimum C_{tot} or d_{opt} along the x-axis. Therefore C_0 can be ignored when doing a cost optimization to determine the β_t [2, 44].

The costs of providing safety C_1 are the costs dependent on d. These costs includes all construction costs (including labour, equipment and material costs) which are dependent on and have an assumed linear relationship with d, and exclude the construction costs which are independent of d (in other words C_0).

The failure costs C_f depend on the probability of failure as a function of the decision parameter $p_f(d)$. The failure costs include: costs of repair (SLS failure), costs related to economic losses, or costs related to societal and environmental consequences [43, 45]. The failure costs considered are those related to both direct and indirect failure consequences. Determining the failure costs is the most important and often most difficult step in cost optimization [19, 46]. The type of failure, in this case SLS failure, will also play a role in determining C_f.

Serviceability failure is defined as the realisation of the action-resistance exceeding the limiting design value. For example if the cracks in a structure are larger than the crack width limit or the deflections are larger than the deflection limit. This is considered to be SLS failure and results in a limited use of the structure and reduction of service life.

3.1.2 Cost Ratios

Due to the complexity in defining failure costs, specifically those related to the indirect consequences, a cost ratio is utilised [1]. This cost ratio C_f/C_1 is a ratio of the failure consequences to the costs of providing safety (per unit of the decision parameter). A relationship (cost ratio) between C_1 and C_f is typically used when doing an economic optimization as the exact costs of the structure are not known, and vary for different structures. This ratio is obtained by setting the derivative of equation 3.2 with respect to d equal to zero as shown in equation 3.4. Where the derivative is equal to zero, d_{opt} may be obtained.

$$\frac{\partial C_{tot}}{\partial d} = C_1 + C_f \frac{\partial p_f(d)}{\partial d} = 0 \tag{3.3}$$

$$1 + \frac{C_f}{C_1}\frac{\partial p_f(d)}{\partial d} = 0 \tag{3.4}$$

While the $\beta_{t,ULS}$ recommended in JCSS were derived from a cost benefit analysis, the $\beta_{t,SLS}$ were derived based on decision analysis and there is no clear link between the above mentioned cost ratios and the SLS specifically for this class of special structures [25]. Previously recommended or investigated ratios are discussed as a guideline for determining ratios for the SLS. Cost ratios for the generic SLS are then discussed.

3.1.2.1 Cost Ratios Previously Investigated

Holicky [17] investigated the influence of the cost ratio, ranging from 1 to 10^6. For the ULS, JCSS [25] defines the cost ratios by 3 consequence classes which can be related to the consequence classes defined in EN1990 [16]. The ratio is defined as $c_{tot} = C_{tot}/C_1$. The three consequence classes in JCSS [25] are minor, moderate and large consequences of failure. Vrouwenvelder [50] defines these cost ratio for JCSS. Sykora et al. [47] gives a range of cost ratios for the various consequence classes for ULS. These ratios are shown in table 3.1.

Table 3.1: Cost Ratios for ULS

Consequence Class	**JCSS [25]**	**Vrouwenvelder [50]**	**Sykora et al. [47]**
CC1 (minor)	$c_{tot} < 2$	$C_f/C_1 = 2$	$1 < C_f/C_1 < 3$
CC2 (moderate)	$2 < c_{tot} < 5$	$C_f/C_1 = 4$	$5 < C_f/C_1 < 20$
CC3 (large)	$5 < c_{tot} < 10$	$C_f/C_1 = 8$	$C_f/C_1 > 20$

3.1.2.2 Cost Ratios for the SLS

A parametric study of the cost ratios is performed to account for a range of consequences of structural failure. The cost ratios from 0.5 up until 100 are considered. The smaller ratios correspond to low costs of failure combined with high costs of increasing safety. The higher ratios correspond to a combination of high consequences of failure and low costs of increasing safety.

The cost ratios C_f/C_1 are discussed based on their consequences. It is left to the engineer's judgement to determine the appropriate cost ratio for his/her specific structure. The following descriptions are general guidelines to assist the engineer in determining what reliability level said structure should have based on its construction and failure costs.

Minor Consequences: $C_f/C_1 = [0.5, 0.8, 1.0]$
These relatively small consequences occur when the failure costs are less than or equal to the costs of providing safety. For example, SLS failure may result in relatively low repair costs C_f compared to the costs of increasing safety C_1 . These low failure costs

typically correspond to minor repair costs with little to no indirect failure costs. It is expected that if serviceability failure was to occur, the use of the structure would not be drastically limited or affected and a small repair effort would easily rectify the structure's integrity.

For these cost ratios a relatively low $\beta_{t,SLS}$ is obtained as serviceability failure does not result in large failure costs. The structural element under consideration might also be an insignificant member of the structure, whose failure would not have a significant impact on the structure's integrity as a whole. Little effort is expected to achieve this level of reliability as the failure consequences are considerably small or even negligible.

For a structure where C_f are significantly lower than C_1, $\beta_{t,SLS}$ might also be negative. When this is the case, ULS will govern the design of the structure and SLS requirements will typically be easily fulfilled.

Moderate Consequences: C_f/C_1= [1.5, 2.0, 4.0]
The failure costs for these consequences are slightly greater than the costs of providing safety and more effort is required to achieve the required level of reliability as the costs of failure now start to increase. These failure costs are related to a more extensive repair effort as failure might result in a significant impact on the use of the structure. The service life of the structure might also be shortened due to this serviceability failure resulting in failure costs associated with the loss of usage of the structure.

For this consequence class, there may be indirect failure costs (for example shortened service life) that are greater than the costs of providing safety. It could also include extensive repair costs which due to the nature of the repairs will cost more than the initial costs of providing safety. For example, societal consequences may play a role for an important water retaining structure or pre-stressed bridge.

Large Consequences: C_f/C_1 = [10.0, 20.0, 50.0, 100.0]
This consequence class refers to a structure or a structural element where the failure costs are significantly (10 or more times) greater than the costs of providing safety.

For example this could be a relatively small element of the structure with relatively cheap costs of providing safety. However, this is a critical element of the structure and failure of this element will result in relatively large consequences of failure with corresponding large failure costs. The direct and indirect failure costs of this element may be significantly larger than the costs of providing safety of this small, critical element. Failure of this element might result in replacing the element and the indirect costs associated with the immediate disuse (temporary or permanent) of the structure for the replacement of the structural element are relatively expensive compared to the initial costs in the construction phase.

3.2 Reliability Analysis

A reliability analysis is performed to obtain the $\beta(d)$ values from the defined limit state function for the SLS.

3.2.1 The Limit State Function

The general SLS equation defined in EN1990 is shown in equation 3.5 [7]. For the SLS, structural failure is defined as the occurrence of the action-resistance effect E exceeding the limiting design value L ($E > L$). E is the random variable that is influenced by the loading but also takes the resistance of the structure into account. For example, the mean value prediction of the maximum deflection $\delta_{m,max}$ is determined from the load effect (action) in combination with the material properties (resistance).

$$g(L, E) = L - E = 0 \tag{3.5}$$

with L Limiting design value

E Action-resistance effect

To perform a first order reliability method (FORM) analysis of equation 3.5, the variables, L and E, need to be assigned values in the generic sense. As L is typically a prescribed limit (for example the crack width limit for cracking or the deflection limit for deflections) it is assumed to be a deterministic value. Based on this assumption, the SLS can be simplified to equation 3.7 by dividing equation 3.5 by L (shown in equation 3.6).

Dividing by L:

$$g(L, E) = \frac{L}{L} - \frac{E}{L} = 0 \tag{3.6}$$

If L is deterministic then:

$$g(1, E) = 1 - \eta E = 0 \tag{3.7}$$

with E SLS action-resistance effect
η Factor which is equal to $1/L$

The range for the factor η includes includes the limiting design values for all SLS scenarios for example crack widths and deflections. For crack widths, the limiting design value is a prescribed value in the range of 0.1 - 1 mm. This results in η values of greater than or equal to one ($\eta \geq 1$). Conversely for deflections, the limiting design value is typically prescribed by the ratio of span over depth or span over 250. As these values are typically larger than one, the resulting η values are less than one ($\eta < 1$). However, η is always greater than zero ($\eta > 0$) as all serviceability limiting design values are assumed to be positive.

Due to the large range of η, it is suitable to assume $\eta = 1$ for the generic case. The action-resistance effect E is then defined for $L = 1$ in the generalised sense. E is defined in equation 3.8 as the product of the predicted action-resistance effect Y and the model factor θ.

$$E = \theta Y \tag{3.8}$$

with θ The model uncertainty
Y The predicted action-resistance effect

θ takes into account all uncertainty associated with the mathematical simplifications used to model physical behaviour by taking into account the statistical differences between the predicted and measured values [10, 20, 36]. Y is the predicted action-resistance effect prescribed by a prediction model (for example in a national standard). For example, it is the mean value prediction of the maximum crack width $w_{m,max}$ for cracking or the mean value prediction of the maximum deflection $\delta_{m,max}$.

Y is random variable typically found through an equation. For example, the crack width equation which comprises of the random variables of both the action effect (loading) and the resistance (material strengths). For consistency, θ must relate the experimental results to the same prediction model used to calculate Y [20].

3.2.1.1 The Mean Value of the Action-Resistance Effect μ_E

It is necessary to assess the reliability index $\beta(d)$ in terms of the decision parameter d. This is done by finding a relationship between d and the random variables of the limit state equation. Typically d adjusts the resistance rather than load for the ULS. This establishes a relationship between the resistance and d. However, for the SLS E is the variable (for example crack widths, deflections, stresses, or vibrations) that is affected by both the action and resistance. A relationship between the mean value of the action-resistance effect μ_E and d needs to be established.

Rackwitz [38] defined a generic decision parameter for the ULS based on the ratio of the mean values of the load effect and the resistance. For the SLS, this can be rewritten as the ratio of the limiting design value μ_L and mean value of the action-resistance effect μ_E. Equation 3.9 shows the relationship used by Rackwitz which is simplified based on the generalised limit state function for the SLS.

$$d = \frac{L}{\mu_E} = \frac{1}{\eta \mu_E} \tag{3.9}$$

where

$$\mu_E = \mu_\theta \mu_Y \tag{3.10}$$

with L Deterministic limiting design value
μ_E Mean value of the action-resistance effect
μ_θ Mean value of the model uncertainty
μ_Y Mean value of the predicted action-resistance effect

From equation 3.9, d is the generic decision parameter that includes any number of physical parameters. These parameters are structural design properties and choices on the SLS performance of the element under consideration. By increasing or changing these parameters, the SLS performance of the element improves. In other words, d is related to the distance between μ_E and 1. As d increases, the distance between μ_E and 1 decreases resulting in a smaller $p_f(d)$. This is illustrated in figure 3.4.

As failure is likely to occur when $d < 1$ ($g < 0$), the range for the decision parameters is chosen to take into account the scenarios where failure is more likely ($d < 1$) and increases until the probability of failure is small ($d = 3$). This range ($d = [0.9 - 3]$) is evaluated in increments of $\Delta d = 0.01$. The small increments are necessary to observe the influence in the change of $\beta(d)$ on the total costs, specifically for a small coefficient of variation V_E as explained in section 3.3.2.

3.2.2 Generic $\beta(d)$ Values

Based on the relationship between the decision parameter and the action-resistance effect, a FORM analysis is performed on the generic limit state function (equation 3.7) to determine $\beta(d)$. The coefficient of variation V_E is a measure of the relative variability of structural behaviour. To account for different structures, V_E is considered to range parametrically. The range considered is $V_E = [0.05 - 0.50]$ in increments of $\Delta V_E = 0.05$. A summary of the variables for the FORM analysis of equation 3.7 is shown in table 3.2. The software, Risk Tools (RT) [41], is used for the FORM analysis.

Table 3.2: Range of Variables Considered in the Generic Reliability Analysis

Variable		Minimum	Maximum	Increments
Decision Parameter	d	0.90	3.00	0.01
Action-Resistance Effect	μ_E	0.33	1.11	$1/d$
	V_E	0.05	0.50	0.05

The action-resistance effect E is assumed to have a lognormal (LN) distribution and is scaled by $\eta = 1$. A LN distribution is chosen as it has a lower bound at zero with a positive skewness and is typically used to describe some loads, material strengths and geometrical data [13]. Most model factors are also assumed to have a LN distribution. A LN distribution is, therefore, deemed suitable to cover the SLS action-resistance effect. This assumption is later verified through the examples in chapters 4 and 5.

The $\beta(d)$ values are related to a one year reference period. Holicky [15] states that the reference period is associated with the period of time corresponding to the statistical assessment of the time variance of the random variables and not with the design working life. It is suitable to determine $\beta_{t,SLS}$ based on a one year reference period. This is supported by Rackwitz [38] by stating:

"The optimal solution for building facilities with or without a systematic rebuilding policy is based on failure intensities and not on time-dependent failure probabilities."

The parametrically varied d and subsequently μ_E in determining $\beta(d)$ is essential for an economic optimisation to determine $\beta_{t,SLS}$. For each V_E a corresponding $\beta_{t,SLS}$ is determined corresponding to d_{opt} where the costs are a minimum. Figure 3.3 shows the influence of V_E and d on the reliability index $\beta(d)$. $\beta(d)$ is directly proportional to d and inversely proportional to V_E.

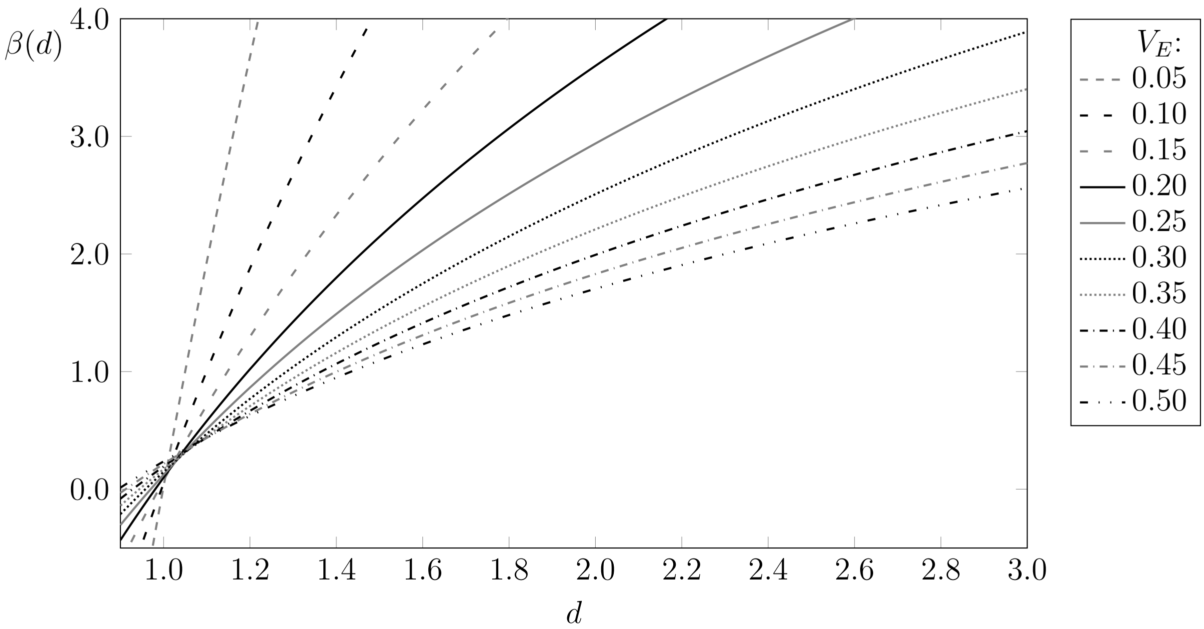
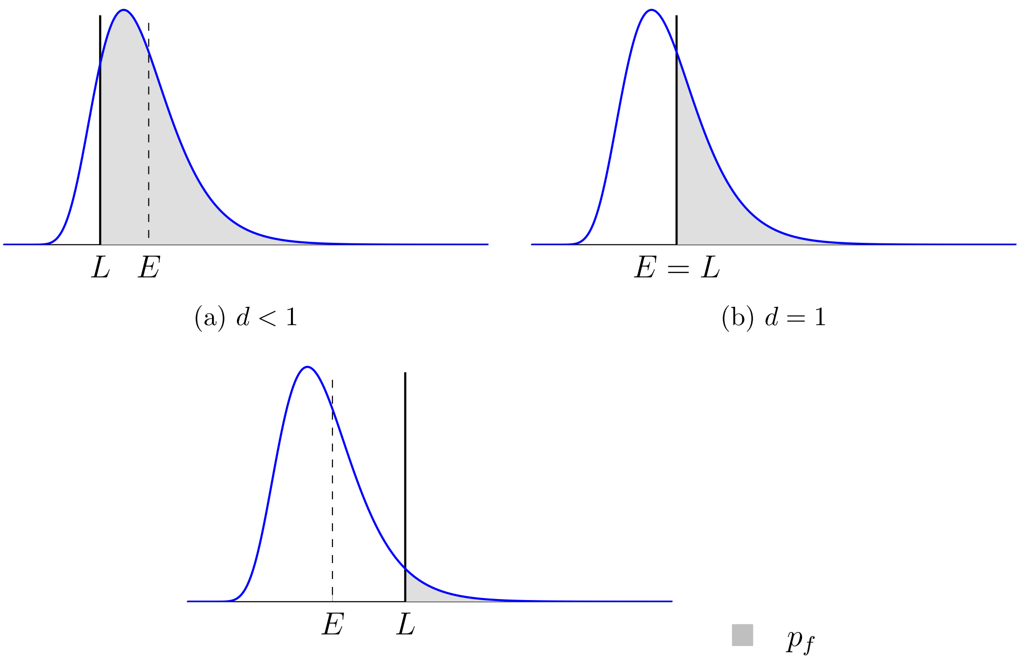


Figure 3.3: Generic $\beta(d)$

The first observation made from figure 3.3 is that as d increases $\beta(d)$ increases as illustrated figure 3.4 where an increase in d results in a decrease in $p_f(d)$. Figure 3.4a illustrates the case for $E > L$ which results in a low $\beta(d)$, this case refers to non-typical design scenarios. Figure 3.4b illustrates the case where $E = L$, and results in $\beta(d) \approx 0$. This corresponds to the current recommended $\beta_{t,SLS}$ for the irreversible SLS. Finally, figure 3.4c illustrates typical design scenarios where $E < L$. As d increases, E decreases resulting in larger $\beta(d)$ values.

The second set of observations made is on the influence of V_E as illustrated in figure 3.5. For a low V_E, the obtained $\beta(d)$ values are high and as V_E increases the $\beta(d)$ values decrease, this is considered to be the normal trend of the influence of V_E. This trend is intuitive as the more uncertain the design, the less reliable the structure is, in other words there is a higher probability of failure. It can be seen that for the range of V_E a large range of $\beta(d)$ values are accounted for.



(a) $d < 1$ (b) $d = 1$ (c) $d > 1$

Figure 3.4: Relationship Between E and L as d Varies

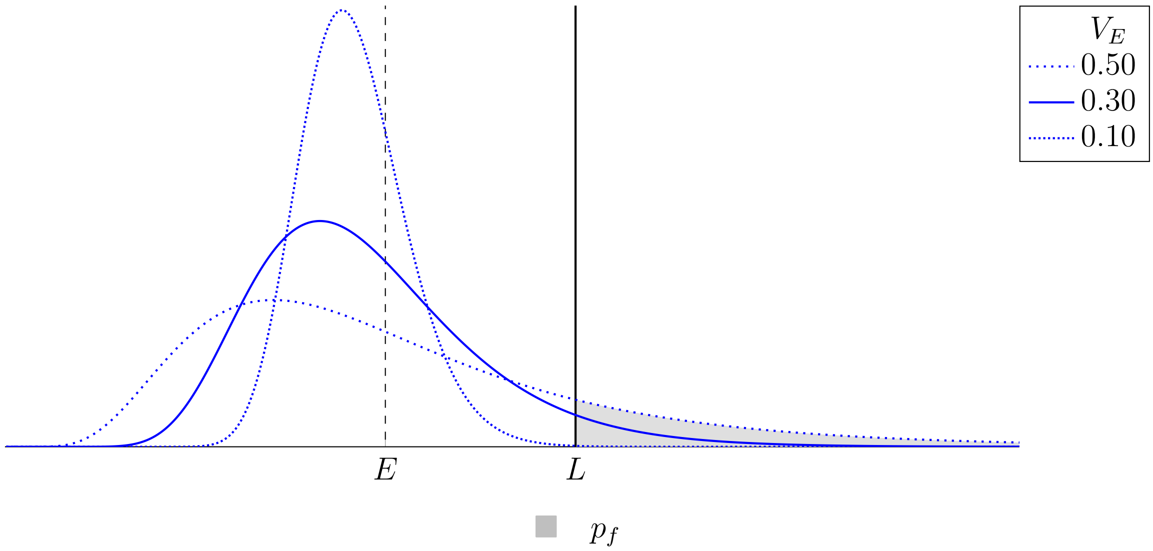


Figure 3.5: Influence of V_E on p_f

However, there is a discontinuity in the trend of $\beta(d)$ values for $d \leq 1$ ($E \geq L$). This is seen clearly in figure 3.6. At $E \approx L$, the $\beta(d)$ values are all approximately equal and before there is very little difference between the obtained $\beta(d)$ values. However for $E \geq L$, the lower V_E the lower the $\beta(d)$ value is which is inverse to what occurs when $g > 0$. This is further explained using figure 3.5 where the failure region is larger for a small V_E and smaller for a large V_E when $d \leq 1$.

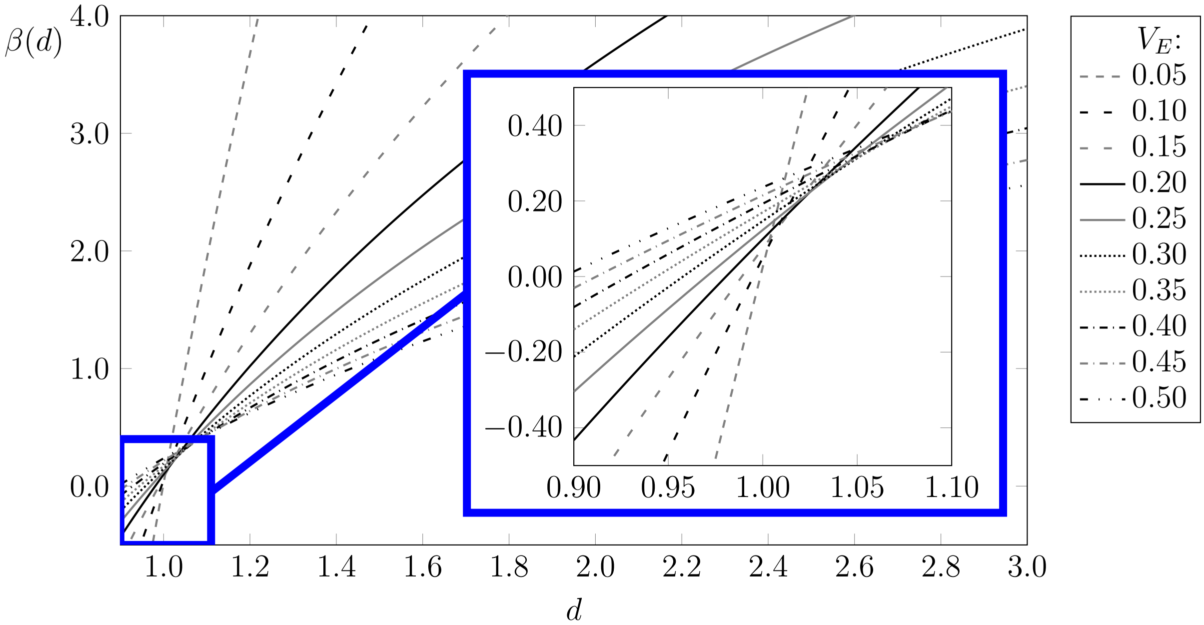


Figure 3.6: Change in $\beta(d)$ at $d \approx 1$

3.3 Target Reliability for SLS

From the range of C_f/C_1 determined in chapter 3.1 and the $\beta(d)$ values for the range of V_E determined in chapter 3.2, a parametric table of the target reliability for concrete structures can be established through economic optimisation. This parametric table takes into account a range of costs and consequence classes while suitably covering all concrete structures (both new and existing) for the SLS (both typical structures and those governed by SLS design).

3.3.1 Cost Optimisation

On the basis of the generalised $\beta(d)$ values, a cost optimisation is performed for the range of cost ratios C_f/C_1. For each combination of C_f/C_1 and V_E, the minimum total cost and corresponding d_{opt} is determined using equation 3.2. $\beta_{t,SLS}$ is taken as $\beta(d_{opt})$ and is summarised in table 3.3. The results from the cost optimisation (C_{tot} versus d) for the parametrically varied V_E and C_f/C_1 are shown in figures 3.7 and 3.8 respectively. These figures are discussed in more detail in the following paragraphs.

Figure 3.7 portrays the influence of C_f/C_1 on C_{tot} for the different values of V_E. See Appendix A figures A.1 to A.10 for the larger figures. The $\beta(d)$ corresponding to V_E is shown on the secondary axis. It can be seen that the optimal decision parameter d_{opt} (where the minimum total costs occur) is dependent on both V_E and C_f/C_1.

For larger values of d ($d > d_{opt}$), C_{tot} is independent of the expected failure costs. This is due to the increased negligible influence of $\beta(d)$ on C_f and the increased cost of providing safety $C_1 d$. In other words for these large values of d the structure is so safe that p_f approaches zero and C_{tot} approaches the construction costs ($C_0 + C_1 d$). However, for smaller values of d ($d < d_{opt}$), C_{tot} varies as C_f/C_1 varies.

The change of magnitude in d_{opt} for different V_E's also increases as the cost ratio increases. For a low C_f/C_1, the minimum total costs occur for a small d as the influence of $\beta(d)$ becomes negligible compared to the costs to improve the reliability versus the small consequences of failure. Whereas for a larger cost ratio, where the expected consequences become more significant, the influence of $\beta(d)$ becomes larger and the position of d_{opt} varies more significantly.

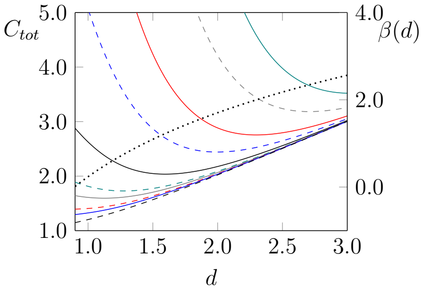
Figure 3.8 portrays the influence of V_E on C_{tot} for different values of C_f/C_1. See Appendix A figures A.20 to A.11 for the full figures. For a specific cost ratio, it can be observed that a lower V_E results in a lower minimum total costs and smaller corresponding d_{opt} compared to those of a higher V_E. However, the corresponding $\beta(d_{opt})$ is higher for a low V_E than for a high V_E. This is also illustrated using figure 3.5.

Figure 3.8e clearly shows the influence of the change in $\beta(d)$ at $d \approx 1$ is apparent even when considering C_{tot}. The general trend is for C_{tot} to be greater for a higher V_E and decrease as V_E decreases. However, the inverse of this trend occurs for $d < 1$ where C_{tot} is greater for a lower V_E. This corresponds to the pivot point identified for the $\beta(d)$ values as shown in figure 3.6. This change in C_{tot} occurs for all cost ratios, but is most apparent on figure 3.8e. For figures 3.8a to 3.8d this pivot in total costs occurs for $C_{tot} > 5$ and exceeds the range of the graph as shown. This inverse in the trend does not typically correspond to design scenarios as $E > L$.

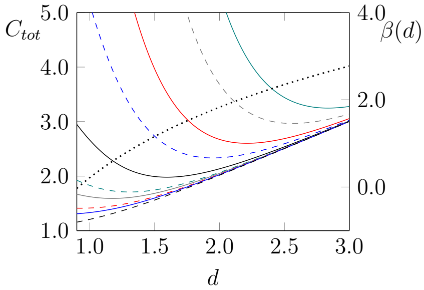
The influence of V_E through $\beta(d)$ on the total costs is apparent in both figures 3.7 and 3.8. This is because the $p_f(d)$ region $(E > L)$ is dependent on V_E. This is shown in figure 3.5 where the p_f for a low V_E is small compared to that of a high V_E. For a low V_E there is a corresponding small variability on C_{tot}. Conversely for a higher V_E the corresponding variability on C_{tot} is greater.

In terms of $\beta_{t,SLS}$ this trend of increasing variability of C_{tot} as V_E increases is identified to have an influence on position of d_{opt} rather than variability $\beta_{t,SLS}$. The corresponding $\beta_{t,SLS}$ decreases as V_E decreases. This is due to the corresponding $\beta(d)$ which varies as V_E varies. From this it can be concluded that the influence of V_E on the total costs is based on the influence of V_E on $\beta(d)$.

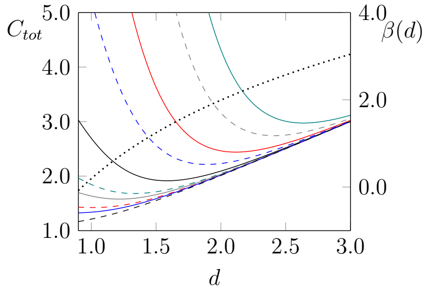
For figures 3.7i and 3.7j, the total costs are not calculated for the entire range of d. This is due to the fact that for such small V_E's, $\beta(d)$ rapidly increases to such a high level of reliability. This results in a negligible p_f ($p_f \approx 0$) for larger values of d. This negligible p_f is clearly illustrated for the low V_E in figure 3.5. However, C_{tot} does increase resulting in a reasonable $\beta_{t,SLS}$ which is not observed at the end of the calculated $\beta(d)$ values.



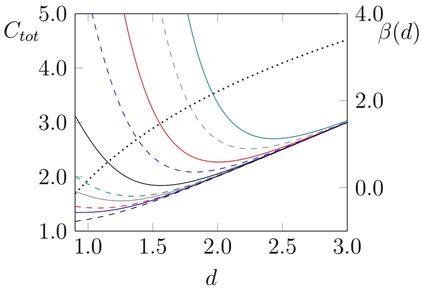
(a) $V_E = 0.50$



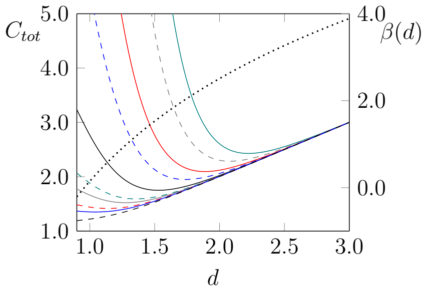
(b) $V_E = 0.45$



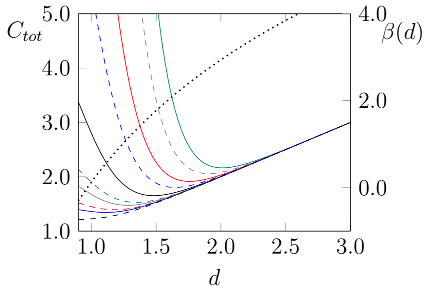
(c) $V_E = 0.40$



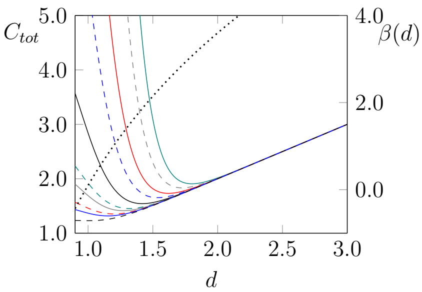
(d) $V_E = 0.35$



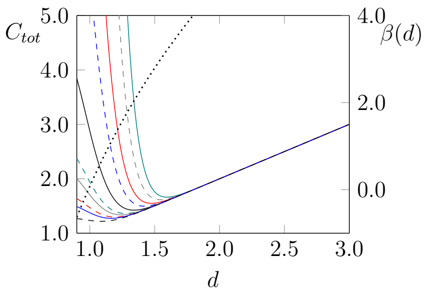
(e) $V_E = 0.30$



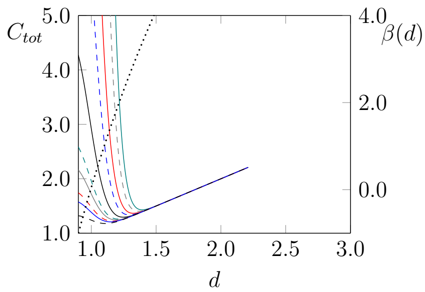
(f) $V_E = 0.25$



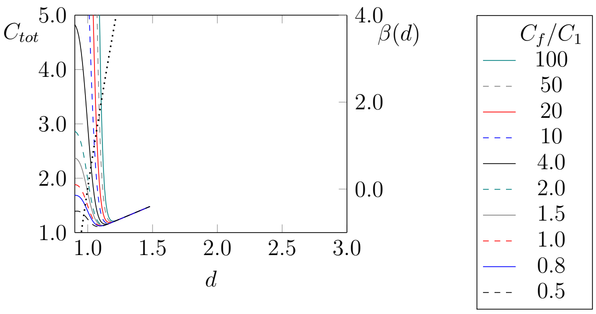
(g) $V_E = 0.20$



(h) $V_E = 0.15$

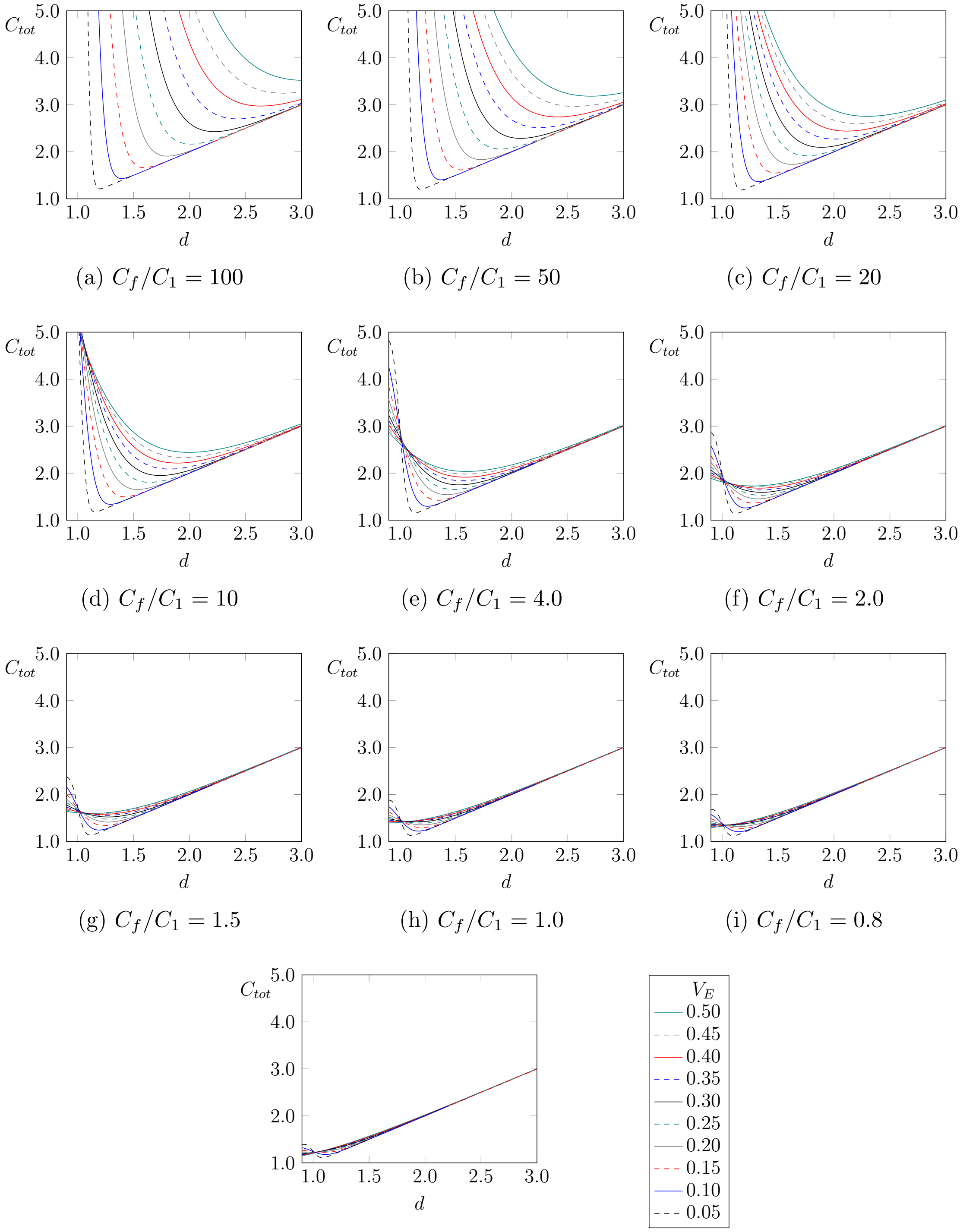


(i) $V_E = 0.10$



(j) $V_E = 0.05$

Figure 3.7: Influence of C_f/C_1 on the Total Costs for Different Values of V_E



(a) $C_f/C_1 = 100$ (b) $C_f/C_1 = 50$ (c) $C_f/C_1 = 20$

(d) $C_f/C_1 = 10$ (e) $C_f/C_1 = 4.0$ (f) $C_f/C_1 = 2.0$

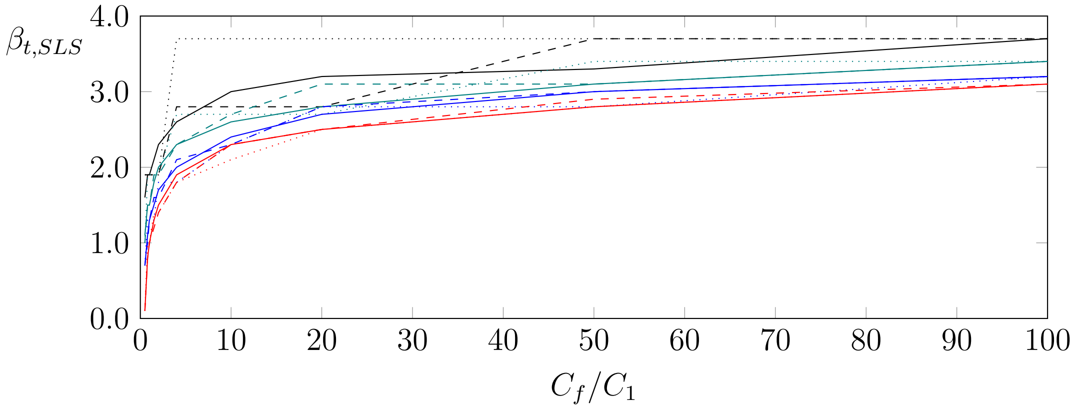
(g) $C_f/C_1 = 1.5$ (h) $C_f/C_1 = 1.0$ (i) $C_f/C_1 = 0.8$

(j) $C_f/C_1 = 0.5$

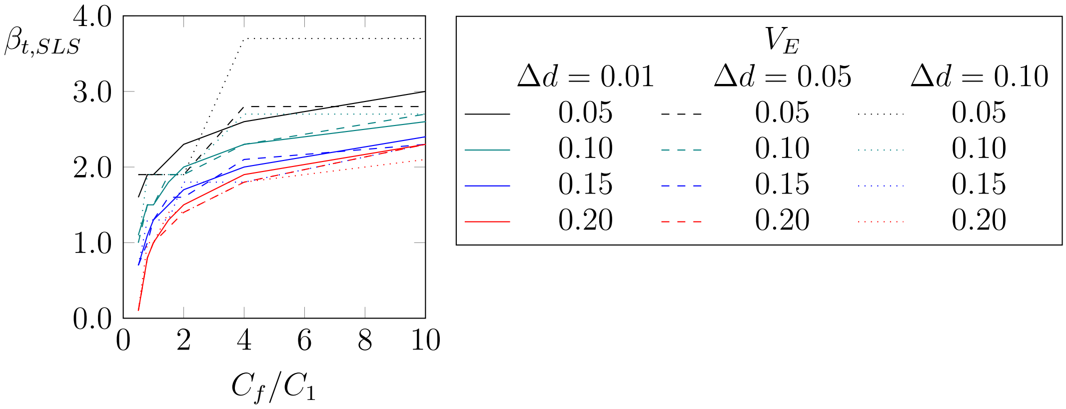
Figure 3.8: Influence of V_E on the Total Costs for Different Values of C_f/C_1

3.3.2 Influence of the Increment of d

In section 3.2.1 the increment of d was chosen to be $\Delta d = 0.01$. The reason for a small increment is based on the steeper slope of $\beta(d)$ as a result of low V_E's. A small investigation into $\beta_{t,SLS}$ was conducted for the range $V_E = [0.05 - 0.2]$ for $\Delta d = 0.01$, 0.05, and 0.10. Figure 3.9 shows the influence of Δd for the range of C_f/C_1 on $\beta_{t,SLS}$.



(a) $C_f/C_1 = 0.5 - 100$



(b) $C_f/C_1 = 0.5 - 10$

Figure 3.9: Influence of Δd

The smaller Δd results in a change of $\beta_{t,SLS}$ as the cost ratio increases, compared to the larger Δd where there is little to no change in $\beta_{t,SLS}$. This is because for a small V_E, d_{opt} is sharply defined due to the steep slope of $\beta(d)$. These large increments in $\beta(d)$ over d result in inaccurate estimates of $\beta(d_{opt})$. This inaccuracy in estimating $\beta(d_{opt})$ is clear especially for $V_E = 0.05$, whereas from $V_E \geq 0.15$ the influence of Δd becomes negligible. However, the difference in $\beta_{t,SLS}$ for $V_E = 0.05$ or 0.10 clearly shows the relationship between $\beta(d)$ and d in determining the total costs and the point where the influence of the construction costs per unit of d becomes significant.

It is evident that for $V_E < 0.15$ the choice of $\Delta d = 0.01$ is necessary to assess the influence of the smaller cost ratios on $\beta_{t,SLS}$. For larger V_E's, it is unnecessary for this increment to be so small as it has no impact on $\beta_{t,SLS}$.

3.3.3 Discussion of the Results ($\beta_{t,SLS}$)

The resulting $\beta_{t,SLS}$ from the economic optimisation are shown in table 3.3. $\beta_{t,SLS}$ systematically increases as the cost ratio increases and V_E decreases. Negative $\beta_{t,SLS}$ values are obtained for very low cost ratios combined with moderate variability V_E.

Table 3.3: $\beta_{t,SLS}$ for a One Year Reference Period

	C_f/C_1									
V_E	**0.5**	**0.8**	**1.0**	**1.5**	**2.0**	**4.0**	**10**	**20**	**50**	**100**
0.50	0.0	0.0	0.0	0.5	0.7	1.2	1.7	2.0	2.3	2.6
0.45	0.0	0.0	0.0	0.6	0.8	1.3	1.8	2.1	2.4	2.6
0.40	-0.1	-0.1	0.2	0.7	0.9	1.4	1.8	2.1	2.5	2.7
0.35	-0.1	0.0	0.4	0.8	1.1	1.5	1.9	2.2	2.6	2.8
0.30	-0.2	0.3	0.6	1.0	1.2	1.6	2.0	2.3	2.6	2.9
0.25	-0.3	0.5	0.8	1.1	1.3	1.7	2.1	2.4	2.7	3.0
0.20	0.1	0.8	1.0	1.3	1.5	1.9	2.3	2.5	2.8	3.1
0.15	0.7	1.1	1.3	1.5	1.7	2.0	2.4	2.7	3.0	3.2
0.10	1.1	1.5	1.5	1.8	2.0	2.3	2.6	2.8	3.1	3.4
0.05	1.6	1.9	1.9	2.1	2.3	2.6	3.0	3.2	3.3	3.7

From an economic perspective, as the relative failure costs increase, a higher $\beta_{t,SLS}$ is recommended. From the perspective of the action-resistance effect, a low V_E results in a higher reliability target compared to a high V_E. This is due to the efficiency of safety measures for a low variability (figure 3.5).

A discontinuity in the observed pattern ($\beta_{t,SLS}$ decreasing as V_E increases) is identified for cost ratios less than one. At $C_f/C_1 < 1$ and $V_E \geq 0.25$, d_{opt} occurs for d less than one (or more simply a non-typical design scenario). When $d < 1$, the inverse in the normal trend for $\beta(d)$ occurs. In other words, a high V_E results in greater $\beta(d)$ than for a lower V_E, as shown in figure 3.3. For $C_f/C_1 < 1$, this pivot in the trend of $\beta(d)$ values and the relatively low value of d_{opt} results in the discontinuity of $\beta_{t,SLS}$. Therefore, for $C_f/C_1 < 1$ a high V_E results in a $\beta_{t,SLS}$ value which is higher than the resulting $\beta_{t,SLS}$ for a medium V_E.

The high target values correspond to high cost ratios or high consequences of failure that are typically associated with ULS failure. For example $\beta_{t,SLS} = 3.7$ shown in table 3.3 corresponds to the $\beta_{t,ULS}$ recommendation in JCSS [25] (table 2.4) for *high* or *moderate* costs of safety measures corresponding to *large* or *minor* consequences of failure respectively. However, for design situations where SLS failure may have severe consequences (for example environmental contamination due to leakage of a hazardous liquid retaining structure) or where the costs of improving safety is low, these values may also be appropriate for SLS design. Another example of severe failure consequence due to SLS failure is the functionality requirements of hospitals and also all buildings expected to function after in emergency operations would have significant failure costs [28].

The low V_E's are expected to correspond to SLS design situations that are governed by deflection or stress requirements, with an presumably small variability. Conversely the higher V_E's are expected to correspond for example to cracking, where model uncertainty is known to be high [30, 31].

Table 3.3 demonstrates that optimal reliability is lower for design situations with higher variability than for lower variability, given a similar cost ratio. This would imply optimal (target) reliability should be lower for SLS design situations that are characterised by high variability, such as crack width predictions, compared to design situations characterised by lower variability such as stresses or deflections. This is because high uncertainty reduces the effect of moderate safety measures (figure 3.5).

3.4 Concluding Remarks

This chapter develops and implements a generic framework for determining $\beta_{t,SLS}$. First, the principle of economic optimisation (used to determine $\beta_{t,SLS}$) is discussed and the components required to perform a cost optimisation are identified. The two cost components are the failure costs C_f and the costs of providing safety C_1. A range of cost and consequence classes are accounted for by the range $C_f/C_1 = [0.5 - 100]$. The limit state equation is generalised as a basis for a reliability analysis. From the generic limit equation, a relationship between the action-resistance effect and the decision parameter is defined. A range of structures is accounted for by the range $V_E = [0.05 - 0.5]$. A FORM analysis of the generic limit state equation is performed to determine $\beta(d)$. Finally cost optimisation is used to determine $\beta_{t,SLS}$.

$\beta_{t,SLS}$ is determined parametrically accounting for a range of structures and a range of cost classes. Table 3.3 covers a large range of structures for a range of serviceability conditions in a generic sense. For a specific structure, the generic framework can be applied to determine $\beta_{t,SLS}$ and chapters 4 and 5 provide such example applications.

4. Example 1: Water Retaining Structure

To show the application of the generic $\beta_{t,SLS}$, the target reliability for the crack width of a water retaining structure (WRS) is determined from the generic development (table 3.3). In order to determine the specific $\beta_{t,SLS}$ for the WRS the action-resistance effect E and its coefficient of variation V_E as well as the cost ratio need to be determined. This is shown in figure 4.1.

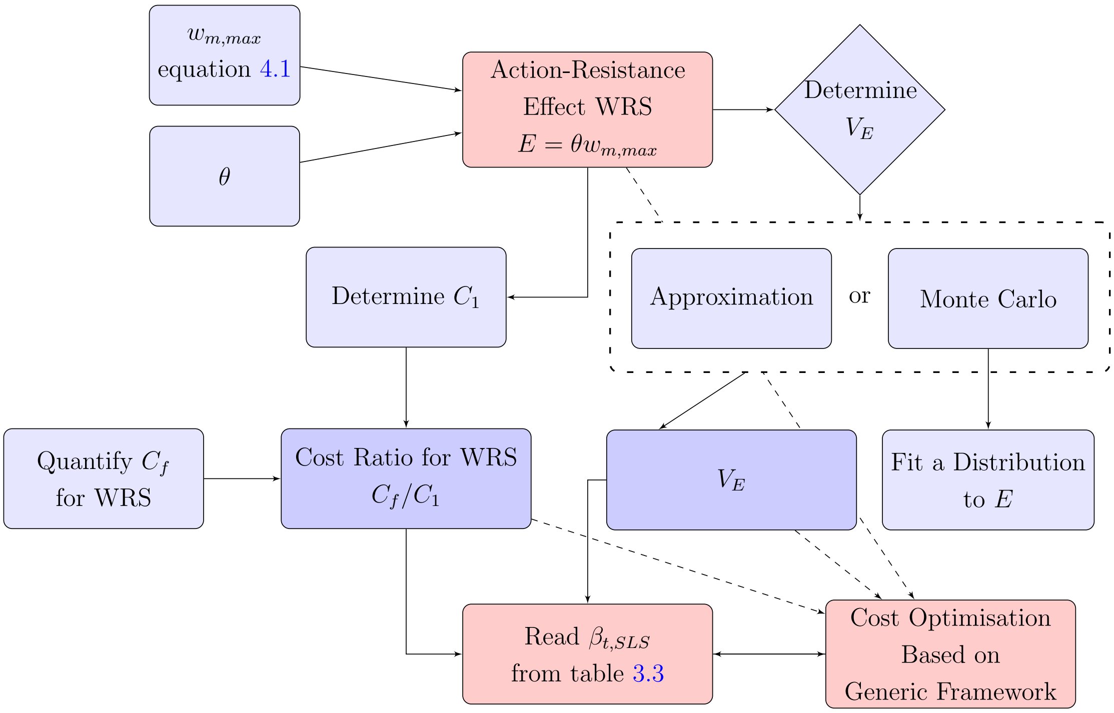


Figure 4.1: Framework - Determining $\beta_{t,SLS}$ for WRS

Figure 4.1 shows the components necessary to obtain $\beta_{t,SLS}$ from table 3.3. First, E needs to be determined. E comprises of the prediction of the crack width $w_{m,max}$ and the corresponding model uncertainty θ. Once E is determined, V_E can be calculated by either Monte Carlo simulation or an approximation of V_E. These two methods are compared to determine how accurately the approximation estimates V_E. The cost ratio is the other component of the generic table of $\beta_{t,SLS}$ values. This is determined by specifically quantifying the failure costs C_f and the costs of providing safety C_1. As shown in figure 4.1, there is a link between E and C_1. Once V_E and C_f/C_1 are quantified for this specific WRS, $\beta_{t,SLS}$ is obtained from table 3.3.

To demonstrate how the $\beta_{t,SLS}$ from the generic structure would compare to the specific structure, cost optimisation is performed. The specific costs and statistical properties of the WRS are used. The results from the cost optimisation are compared to $\beta_{t,SLS}$ read from the table.

4.1 Action-Resistance Effect

From the generalised limit state equation (equation 3.7) the action-resistance effect is defined as $E = \theta Y$ (equation 3.8). The model uncertainty θ for the crack width prediction model of the Eurocode is discussed in section 4.1.2. For cracking the predicted serviceability condition Y is the mean value prediction of the maximum (mean-maximum) crack width $w_{m,max}$ of the WRS. $w_{m,max}$ is based on the Eurocode as discussed in section 4.1.1.

Two circular water retaining structures (WRS) are considered for both flexural and tensile crack widths according to the specifications of the Eurocodes. EN1992 Part 3 [9] deals with WRS. Limitations on the crack width prediction of the structure typically govern the design of WRS. Although the SLS governs the design, the structure must still be checked to ensure that ULS requirements are met. It is assumed that the ULS requirements are met and serviceability governs the design as the focus of this study is to determine $\beta_{t,SLS}$.

4.1.1 Crack Width Equation

The mean-maximum crack width for the WRS is calculated based on the prediction model of the Eurocode. EN1992-1-3 [9] stipulates the provisions for retaining structures and the crack width equation is found in EN1992-1-1 [8]. As a reliability analysis is based on a mean value prediction, $w_{m,max}$ is calculated from the mean values of the random variables which the crack width equation consists of. $w_{m,max}$ is calculated using equation 4.1, which follows the provisions of section 7.3.4 in

EN1992-1-1 [8].

$$w_{m,max} = s_{rm,max}(\varepsilon_{sm} - \varepsilon_{cm}) \tag{4.1}$$

with $s_{rm,max}$ Maximum crack spacing (equation 4.4).
ε_{sm} Steel strain (equation 4.2).
ε_{cm} Concrete strain (equation 4.3).

The crack width is the mean-maximum crack spacing $s_{rm,max}$ multiplied by the difference between the mean strain in the steel ε_{sm} and the mean strain in the concrete ε_{cm}. The strains in the steel and concrete are determined by equations 4.2 and 4.3 respectively. The difference between the strains must be greater than $0.6\sigma_s/E_s$. However, as the reliability analysis is based on the mean value prediction and not the design value the crack width prediction in this chapter ignores the limits on the design. $s_{rm,max}$ is calculated by using equation 4.4 and is a mean value prediction of the maximum crack spacing.

$$\varepsilon_{sm} = \frac{\sigma_s}{E_s} \tag{4.2}$$

$$\varepsilon_{cm} = \frac{k_t \frac{f_{ctm}}{\rho_{p,eff}}(1 + \alpha_e \rho_{p,eff})}{E_s} \tag{4.3}$$

$$s_{rm,max} = k_3 c + k_4 k_1 k_2 \frac{\phi}{\rho_{p,eff}} \tag{4.4}$$

with E_s Elastic modulus of steel.
σ_s Stress in tension reinforcement, assuming a cracked section. $\sigma_s = \frac{M_s}{zA_s}$ or $\sigma_s = \frac{T_s}{A_s}$
k_t The factor dependent on the load duration. See table 4.1.
f_{ctm} Mean value of the tensile concrete strength.
$\rho_{p,eff}$ Ratio of $(A_s + \xi_1 A_p)/A_{c,eff}$.
A_s Area of reinforcing steel.
A_p Area of pre- or post tensioned tendons.
$A_{c,eff}$ Effective area of concrete surrounding the reinforcement in tension.
ξ_1 Adjusted ratio of the bond strength.

α_e	Ratio of E_s/E_{cm}.
E_{cm}	Concrete modulus.
c	Concrete cover.
ϕ	The bar diameter.
k_1	The factor taking the bond properties into account. See table 4.1.
k_2	The factor taking the distribution of strain into account. See table 4.1.
k_3	See table 4.1.
k_4	See table 4.1.

For $\rho_{p,eff}$, A_s is the area of steel reinforcement, and A_p is the area of prestressing tendons. If there are no prestressing tendons in the structure $A_p = 0$. $A_{c,eff}$ is the effective depth in tension $h_{c,eff}$ multiplied by the width b. The effective depth $h_{c,eff}$ is the minimum of d^*, $h/2$, and $2.5(h-d^*)$. Where d^* is the depth to the reinforcement.

The final input $w_{m,max}$ equation is summarised in equations 4.5 and 4.6 for flexure F and tension T respectively.

$$w_{m,max}(F) = [3.4c + 0.425k_1k_2(\frac{\phi h_c b}{A_s})] \times [\frac{M_s}{zA_sE_s} - \frac{k_t f_{ctm} h_c}{A_s}\frac{1+\alpha_e A_s/(h_c b)}{E_s}] \quad (4.5)$$

$$w_{m,max}(T) = [3.4c + 0.425k_1k_2(\frac{\phi h_c b}{A_s})] \times [\frac{T_s}{A_sE_s} - \frac{k_t f_{ctm} h_c}{A_s}\frac{1+\alpha_e A_s/(h_c b)}{E_s}] \quad (4.6)$$

Table 4.1: "k" Factors Used in the Crack Width Prediction [8]

Factor	Value	Details
k_t	0.6	Short term loading
	0.4	Long term loading
k_1	0.8	High bond bars
	1.6	Bars with effectively plain surface
k_2	0.5	Bending
	1.0	Pure tension
k_3	3.4	
k_4	0.425	

The factor for the load duration k_t defines whether short term (ST) or long term (LT) cracking occurs. However, it is not explicitly defined as what a ST load duration versus a LT load duration is. It can be assumed in practice that LT loading refers to the load duration of a load that is permanent or occurs frequently/regularly throughout the structures service life. ST loading, on the other hand, is for example the load case if the structure is to be drained for repairs and there is no longer any water pressure. ST loading also refers to the initial load duration of the water pressure before the effects of creep are taken into account. Another influence on LT cracking is the concrete modulus E_{cm}. For long term cracking E_{cm} should be taken as the effective long term modulus which takes creep into account [35].

From the "k" factors, four different crack widths need to be predicted. The crack widths are defined based on the type of loading (flexure or tension) which is determined by the factor k_2. The type of loading also influences the prediction of the steel stress (vertical wall steel stress or hoop stress). The two load types are then split by k_t into ST and LT loading. These four predictions are all further investigated to determine whether short or long term loading governs. They are summarised in table 4.2.

Table 4.2: Crack Width Prediction Scenarios

Name	Loading	Load Duration
F-LT	Flexure	Long Term
F-ST	Flexure	Short Term
T-LT	Tension	Long Term
T-ST	Tension	Short Term

4.1.2 Model Uncertainty of Crack Width

The model uncertainty θ for the EN1992 crack width prediction determined by McLeod et al [32, 33] is used. McLeod et al [32, 33] determined θ for EN1992 cracking based on the available experimental data in literature. These model factors are shown in table 4.3.

Table 4.3: Model Uncertainty for Concrete Crack Models [32, 33]

	μ_θ	V_θ	PDF
F-LT	1.416	0.338	N
F-ST	1.164	0.507	LN
T-LT	0.900	0.302	N
T-ST	0.862	0.227	N

From table 4.3, it is evident that the prediction model of EN1992 better estimates the tensile case. The tensile case has a conservative bias, while the flexural case has an unconservative bias (under predicts). The F-LT case is largely underestimated by EN1992. McLeod et al [32] attributed this to the fact that the EN1992 prediction model does not consider the effects of shrinkage.

Compared to the other model factors, only θ for F-ST has a lognormal (LN) distribution, which corresponds to the previous assumptions as shown in table 2.1. This model uncertainty also had the largest sample size for determining θ which could be an influencing factor on the differences in distribution. However McLeod et al [33] concluded that further investigation into the reason for the LN or N distribution would be necessary.

As the random variables of the prediction model for the crack width Y are all multiplied by θ, the influence of the distribution type (LN or N) of θ needs to be investigated. Therefore, θ is analysed for both a N and LN distribution in this chapter. By doing so, the assumption of E having a LN distribution in the generic case can be investigated if E for the WRS does or does not have a LN distribution.

4.1.3 Prediction of the WRS Action-Resistance Effect

The action-resistance effect E of the WRS is determined for when $E = L$ (or in this case when $\mu_\theta w_{m,max} = w_{lim}$). The decision parameter for this example is chosen as a unit of A_s [42]. A_s is varied to determine where the predicted crack width is equal to the crack width limit w_{lim} ($A_s = A_{s_L}$). The crack width is assumed to be limited to $w_{lim} = 0.2mm$ and correlates to a tightness class of 1 which allows for little to no leakage.

The mean-maximum crack width of two WRS (water retaining structures) is calculated according to EN1992 for both flexural and tensile cracking. Short and long term cracking are investigated to determine which one governs. The amount of reinforcement A_{s_L} for the four scenarios is determined so that $\mu_E = w_{lim}$. The two WRS have different thickness's, bar diameters, and covers to investigate the influences of these parameters on $\beta_{t,SLS}$. The two water retaining structures are named WRS 1 and WRS 2.

The sectional and material properties of the WRS are summarised in table 4.4. Both WRS are designed to have a volume of $V = 1500m^3$. The dimensions are defined as: height $H = 5m$, diameter $D = 20m$, and thickness $h = 500mm$ or $350mm$ for WRS 1 and 2 respectively. The diameter of the structure has no influence on the flexural design as the crack width is determined per meter strip ($b = 1m$). However for tension, the diameter of the WRS is used to determine the hoop tension in the wall.

Table 4.4: WRS Section and Material Properties

Variable	Symbol	WRS1	WRS 2	Units
Diameter	D	20.0	20.0	m
Height	H	5.0	5.0	m
Width	b	1.0	1.0	m
Wall Thickness	h	500.0	350.0	mm
Cover	c	40.0	30.0	mm
Diameter	ϕ	20.0	25.0	mm
Concrete Tensile Strength	$f_{c,tm}$	2.6	2.6	MPa
Concrete Modulus	$E_{c,ST}$	31.0	31.0	GPa
Elastic modulus of steel	E_s	200.0	200.0	GPa
Density of Water	γ_w	10.0	10.0	kN/m^2
Area of reinforcement	A_{s_L}	varies*		mm^2

* varies due to μ_θ to determine $\mu_E = w_{lim}$

The effect of the water is the main load on the structure. The density of the water is $\gamma_w = 10kN/m^2$ and is considered as a quasi permanent load. Concrete with a compressive strength of $f_{cu} = 30MPa$ ($f_{cm} = 38MPa$) and reinforcement with a yield strength of $f_y = 450MPa$ ($f_{ym} = 500MPa$) are used for the calculations. Any other values required to determine $w_{m,max}$ and the relevant equations used are summarised in table 4.5.

Table 4.5: Additional Variables and the Relevant Equations

Variable	Symbol	Equation	WRS1	WRS 2	Units
Moment	M_s	$\gamma_w H^3/6$	208.3	208.3	kNm
Tension Force	T_s	$\gamma_w HD/2$	500.0	500.0	kN
Depth to reinforcement	d^*	$h - c - \phi/2$	450.0	307.5	mm
Modular ratio *	$\alpha_{e,LT}$	$(E_s/E_{cm,LT})$	15.0	15.0	
	$\alpha_{e,ST}$	$(E_s/E_{cm,ST})$	6.5	6.5	
Moment lever arm **	z	$0.9d^*$	315	276.8	mm
Effective depth	$h_{c,eff}$	$\min[h/2; 2.5(h-d^*); d^*]$	125.0	106.3	mm
Effective area in tension	$A_{c,eff}$	$bh_{c,eff}$	125000.0	106250.0	mm^2
Steel stress	σ_s	$M_s/(zA_s) = T_s/A_s$	varies with A_s		MPa

$*$: assumed ratio which takes $E_{cm,LT}$ into account

$**$: simplified expression to determine the lever arm

Some simplifications are made on the $w_{m,max}$ prediction. The first assumption is that the lever arm acts at $0.9d^*$. This is a simplification of the calculation of the lever arm to simplify the calculations of the Monte Carlo Simulation and the FORM analysis. This simplification is not far from the value of z obtained in the calculation therefore making it a reasonable assumption. The second simplification made is on the modular ratio $\alpha_{e,LT}$ which is assumed to be $\alpha_{e,LT} = 15$ for long term cracking to take into account the effect creep has on the concrete modulus $E_{cm,LT}$ [34].

A_{s_L} for $\mu_E = w_{lim}$ is determined for each cracking scenario, and the calculations are shown in appendix B. A_{s_L} varies for the four scenarios based on the model factor θ, the load type k_2, and the load duration k_t. LT cracking governs for both flexure and tension. The results are shown in table 4.6 along with μ_θ and $w_{m,max}$ for each scenario. F-LT cracking requires the most reinforcement to achieve the prescribed limit for both WRS 1 and 2. This is mainly due to the high unconservative bias of the model uncertainty.

It is observed from table 4.6 that for tension, there is no difference in A_{s_L} to obtain the limit. The main reason for this is that the depth of the section h has a negligible influence on the steel stress of the section.

Table 4.6: Amount of Reinforcement A_{s_L} for $E = w_{lim}$

	A_{s_L} (WRS 1) mm^2	A_{s_L} (WRS 2) mm^2	θ	$w_{m,max}$ mm	E mm
F-LT	3184	4244	1.42	0.14	0.20
F-ST	2568	3615	1.16	0.17	0.20
T-LT	3026	3057	0.90	0.22	0.20
T-ST	2698	2796	0.86	0.23	0.20

As E is governed by both F-LT and T-LT, $\beta_{t,SLS}$ will be governed by either F-LT or T-LT. Therefore, it is necessary to further investigate only these two scenarios in determining $\beta_{t,SLS}$ for both WRS. From the crack width equations (equations 4.5 and 4.6), the random variables can be assigned statistical properties. From these random variables the coefficient of variation for the crack width and the action resistance effect can be determined.

4.2 Generic $\beta_{t,SLS}$ for the WRS Obtained from the Generic Table

This section shows how to obtain the generic $\beta_{t,SLS}$ for the WRS from the generic development of chapter 3. First the coefficient of variation of the WRS is computed for both F-LT and T-LT. Next the cost ratio for the WRS is quantified. From these specific values for the WRS, $\beta_{t,SLS}$ can be obtained.

4.2.1 Calculation of V_E for the WRS

The coefficient of variation of the action-resistance effect V_E is calculated to determine $\beta_{t,SLS}$ from table 3.3. As E consists of random variables with their own statistical parameters, V_E needs to be calculated from these random variables. Two methods are used to determine V_E. First, Monte Carlo is used to accurately determine $V_{E_{MCS}}$ and also the distribution of E. Next an approximation of $V_{E_{APP}}$ is done and compared to the results of the Monte Carlo simulation.

4.2.1.1 Monte Carlo Simulation

A brief overview of Monte Carlo Simulation is provided. The Monte Carlo simulation is then performed on E of the two WRS to determine $V_{E_{MCS}}$.

The Monte Carlo simulation (MCS) technique is based on the random sampling of variables [13]. The MCS is a simple random sampling method or statistical trial method that makes realizations based on randomly generated sampling sets for uncertain variables [39].

Choi et al [5] and El-Reedy [39] both set out the simple MCS procedure through the following steps. These steps are simply shown in figure 4.2.

1. The first step is to select the distribution type(s) for the random variable(s). Each random variable is described by three statistical parameters: mean, standard deviation and distribution type. The input is shown in figure 4.2 by the distribution type, however, it the mean and standard deviation of that random variable.

2. The next step is to generate a sampling set from the distribution(s) for the chosen sample size. A sampling set is generated for the action-resistance effect of the WRS. A large sample size is chosen based on the mathematical theory that sample averages tend to stabilize as the sample increases [6].

3. The final step is to conduct simulations using the generated sampling set. The generated sample set, or in this case $w_{m,max}$, has a mean and standard deviation while distribution type can be determined from the frequency distribution.

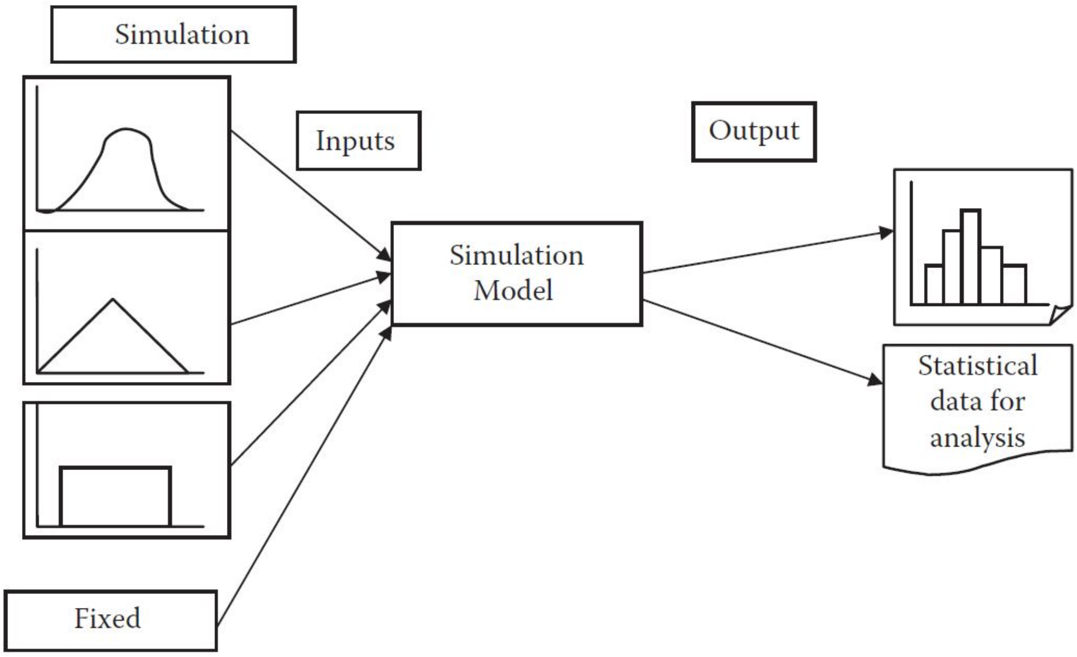


Figure 4.2: Monte Carlo Simulation [39]

The Monte Carlo Simulation (MCS) is performed using the trial version of the @RISK [40] software which is an Excel add-in. From the MCS, the coefficient of variation $V_{E_{MCS}}$ and distribution of the action-resistance effect can be determined.

The random variables that the crack width comprises of and the model uncertainty are the input variables for the MCS. The random variables and their statistical properties for the MCS are shown in table 4.7. It is assumed that the statistical properties (V_i and PDF) for WRS 1 and 2 are the same. The assumptions for the random variables are made based on what is currently available in literature. From table 4.7, the probability density function (PDF) of the variables is either deterministic (det), normal (N), or lognormal (LN).

In table 4.7, the amount of reinforcement is defined as A_{s_L}. This is because A_{s_L} required such that $\mu_E = w_{lim}$ is different for the two structures under consideration.

Table 4.7: Statistical Parameters of the WRS

Variable		Units	PDF	μ_i	V_i	Ref
Moment	M_s	kNm	N	208.0	0.05	[14, 18]
Tension	T_s	kN/m	N	500.0	0.05	[14, 18]
Wall Height	H	m	N	5.0	0.01	[18]
Wall Diameter	D	m	N	20.0	0.01	[18]
Width	b	m	det	1000.0	0.00	[14, 35]
Wall Thickness	h	mm	LN	500.0/350.0	0.01	
Cover	c	mm	LN	40.0 /30.0	0.15	[35]
Diameter	ϕ	mm	det	20.0/25.0	0.00	[14, 25, 30]
Elastic modulus of steel	E_s	GPa	det	200.0	0.00	[14, 25, 35]
Concrete Tensile Strength	f_{ctm}	MPa	LN	2.6	0.10	[35]
Strength Factors	k_1, k_2, k_3, k_t		det	k	0.00	[14, 30]
Reinforcement Area	A_{s_L}	mm^2	N	A_{s_L}	0.02	[18, 27]
Model Uncertainty	F-LT θ		N/LN	1.4	0.34	[32]
	T-LT θ		N/LN	0.9	0.30	[33]

For WRS 1, $A_{s_L} = 3200mm^2$ and $3000mm^2$ for F-LT and T-LT respectively. Similarly for WRS 2, $A_{s_L} = 4300mm^2$ and $3000mm^2$ for F-LT and T-LT respectively.

From figure 4.2, the values in table 4.7 are entered as the inputs, @RISK is used as the simulation model, and the outputs obtained from @RISK are further discussed in this section.

The results of the output of the MCS using @RISK are summarised in table 4.8. The coefficient of variation was determined for both $w_{m,max}$ and E. As the statistical properties of WRS 1 and WRS 2 are assumed to be the same, both $V_{E_{MCS}}$ and $V_{w_{m,max}}$ for the two WRS are the same. The influence of V_θ on $V_{E_{MCS}}$ is evident in the results. Therefore the importance of choosing the correct model factor should not be overlooked. The distribution (N or LN) of θ also has no influence on $V_{E_{MCS}}$

and only on the distribution of E.

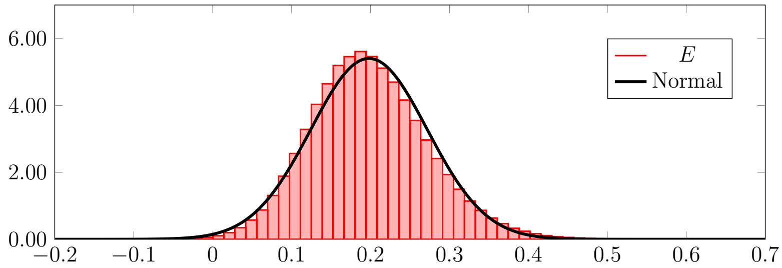
Table 4.8: Results from MCS

	PDF θ	Variable	WRS 1 PDF	WRS 1 μ $[mm]$	WRS 1 V	WRS 2 PDF	WRS 2 μ $[mm]$	WRS 2 V
F-LT	-	$w_{m,max}$	LN	0.140	0.150	LN	0.138	0.151
T-LT	-	$w_{m,max}$	LN	0.225	0.131	LN	0.216	0.126
F-LT	N	E	N	0.198	0.373	LN	0.196	0.373
F-LT	LN	E	LN	0.198	0.373	LN	0.196	0.374
T-LT	N	E	N	0.202	0.332	N	0.194	0.330
T-LT	LN	E	LN	0.202	0.332	LN	0.194	0.330

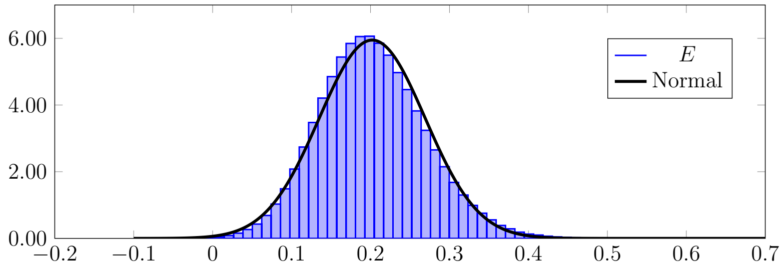
The distribution of E for the two structures (F-LT and T-LT) is compared to a normal and lognormal distribution using @RISK. This is shown in figures 4.3 and 4.4. It is observed in figures 4.3 and 4.4 that the distribution of θ (N or LN) governs the distribution of E. For θ having a N distribution, a LN distribution could only be fit to the histogram of E for WRS 2 F-LT. However, it is still reasonable to conclude that the distribution of θ governs the distribution of the WRS for this specific example. By changing any of the random variable's statistical parameters, the distribution of E may no longer be governed by the distribution of θ. A reduced V_θ would also have the same effect.

The normal distribution of E for the WRS is different from the generic E which is assumed to have a lognormal distribution. The influence of this is further investigated through a full cost optimisation for the cost ratio of the WRS as discussed in section 4.3.

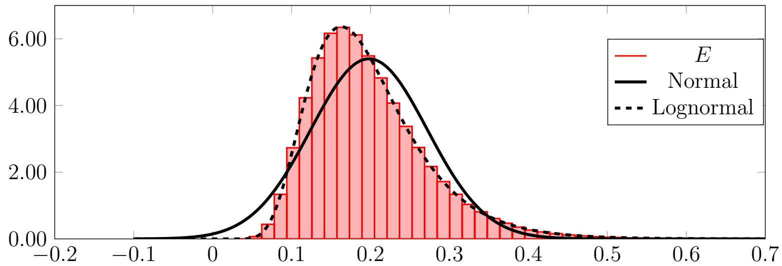
From the results shown in table 4.8, the influence of V_θ and $V_{w_{m,max}}$ is observed. Both V_θ and $V_{w_{m,max}}$ for tensile cracking are less than V_θ and $V_{w_{m,max}}$ for flexural cracking. This results in a smaller V_E for tension. From this it is expected that tensile cracking has a larger $\beta_{t,SLS}$ if the cost ratio is the same for flexure and tension.



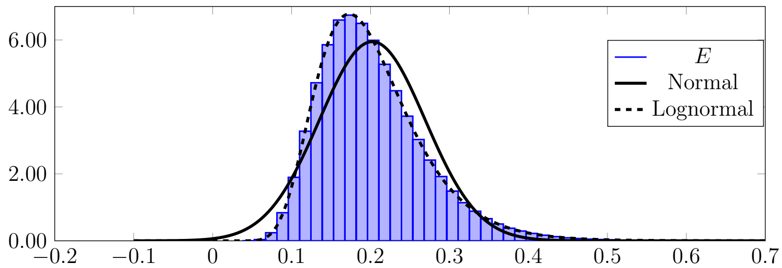
(a) Flexure (θ-N)



(b) Tension (θ-N)

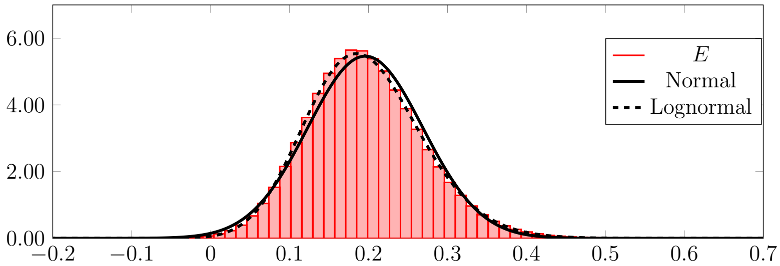


(c) Flexure (θ-LN)

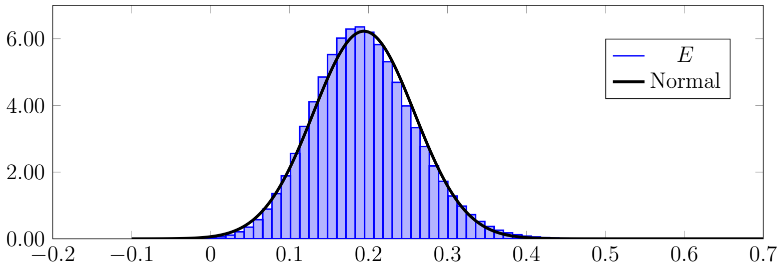


(d) Tension (θ-LN)

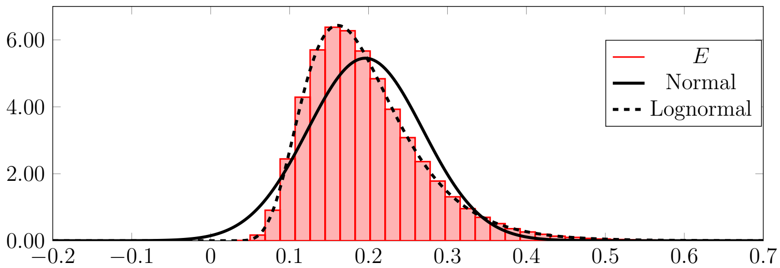
Figure 4.3: Histogram of the Action-Resistance Effect E for WRS 1 for Differentiated Assumed Distribution Types of the Associated Model Factor θ



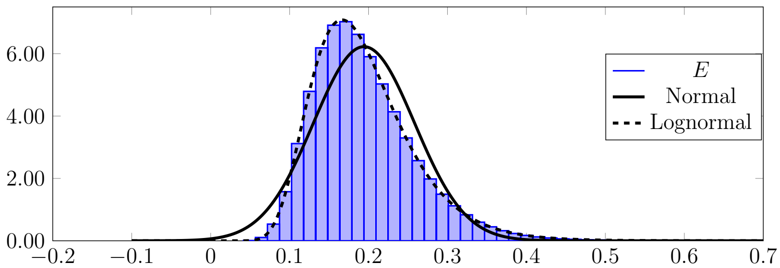
(a) Flexure (θ-N)



(b) Tension (θ-N)



(c) Flexure (θ-LN)



(d) Tension (θ-LN)

Figure 4.4: Histogram of the Action-Resistance Effect E for WRS 2 for Differentiated Assumed Distribution Types of the Associated Model Factor θ

4.2.1.2 Approximation of V_E

It is not always possible to determine V_E through MCS or any other sampling method. If a quick estimate of V_E is required, a complete MCS of a sufficient sample size is time consuming. It is, therefore, necessary to consider an approach which quickly approximates V_E. A simplified method of determining $V_{E_{APP}}$ based on the coefficient of variation of the random variables V_i is conducted to compare these results to the sampling analysis. For assumed normally distributed uncorrelated variables $V_{E_{APP}}$ is determined through equation 4.7 [13, 36].

$$V_{E_{APP}} = \sqrt{\sum V_i^2} \tag{4.7}$$

with $V_{E_{APP}}$ Coefficient of variation of the action-resistance effect.
V_i Coefficient of variation of the random variable i.

For the WRS, the 5 random variables identified are area of reinforcement, cover, loading, concrete tensile strength and model uncertainty. V_i of the random variables is obtained from table 4.7. $V_{E_{APP}}$ is the approximated for both flexure and tension. The results for the approximation are shown in table 4.9 along with the results from the MCS.

Table 4.9: Comparative Table of V_E for the WRS

	MCS	APP
	$V_{E_{MCS}}$	$V_{E_{APP}}$
F-LT	0.373	0.387
T-LT	0.332	0.356

APP = approximation (equation 4.7)

As seen in table 4.9, the approximated $V_{E_{APP}}$ is reasonably close to $V_{E_{MCS}}$ ($\Delta V_E \approx 0.025$). It is therefore a reasonable method for determining V_E. Although this method is quicker and yields similar results there is one major drawback to using the simplified approach. The distribution of E cannot be determined for the simplified calculation of $V_{E_{APP}}$. However, if the assumption in the generic case of a LN distribution holds for a non-lognormally distributed E, it may be reasonable to conclude that this method is sufficient.

4.2.2 Calculation of the Costs

To obtain $\beta_{t,SLS}$ from table 3.3, the specific cost ratios C_f/C_1 also need to be quantified. This is done by determining the specific costs of providing safety C_1 and the specific failure costs C_f. The costs in this section are a simple representation of some of the cost considerations when determining C_f/C_1. The assumptions made for these costs are stated in the respective subsections (section 4.2.2.1 and 4.2.2.2). As a consistent unit of measurement is necessary, the costs for the two WRS are calculated as Rand per meter squared (R/m^2).

4.2.2.1 Costs of Providing Reinforcement C_1

For the generic structure, the cost of providing safety is equal to the cost of one unit of d. Since $d = 1$ implies that $E = L$ for the generic structure (equation 3.9), the cost of providing reinforcement C_1 for the WRS must correlate to $\mu_E = w_{lim}$. For the WRS, the decision parameter is the amount of reinforcement A_s. Therefore C_1 is the cost of providing A_{s_L} such that $\mu_E = w_{lim}$. A_{s_L} is the area of reinforcement required such that $\mu_E = w_{lim}$. It is assumed that all other construction costs C_0 are independent of A_{s_L}.

To determine C_1 for the WRS, the cost per reinforcement bar is determined. The cost to provide one bar of reinforcement is quoted as $R10.20/kg$ [22]. The weight of one bar (kg/m) is determined using equation 4.8. One Y20 (WRS 1) and Y25 (WRS 2) weigh $w_s = 2.47kg/m$ and $3.85kg/m$ respectively. C_1 is then determined using equation 4.9 as the cost per bar (R/kg) times the weight of the bar (kg/m) times the number of bars per meter A_{s_L}/A_{s1} ($/m$). The cost C_1 for the two WRS is summarised in table 4.10.

$$w_s = \rho \frac{\phi^2 \pi}{4} \tag{4.8}$$

$$C_1 = \frac{A_{s_L}}{A_{s1}} \times w_s \times 10.20 \tag{4.9}$$

with ρ Density of steel $\rho = 7850kg/m^3$

ϕ Bar diameter

A_{s_L} Area of reinforcement which results in $E = w_{lim}$

A_{s1} Area of reinforcement of one bar

Table 4.10: Cost of Providing Safety C_1 for the WRS

		A_{s_L} mm^2	C_1 R/m^2
WRS 1	Y20	314	25.19
	F-LT	3200	256.62
	T-LT	3000	240.58
WRS 2	Y25	491	39.30
	F-LT	4300	344.30
	T-LT	3000	240.21

4.2.2.2 Failure Costs C_f

Quantifying the failure costs is considered to be the most difficult step in terms of cost optimisation as both direct and indirect failure consequences need to be considered [47]. For this example, the failure costs C_f are simplified to account only for the failure costs associated with providing water tight lining for an already cracked concrete section. Some of the other failure costs which may occur for a similar WRS are indicated below:

- Labour costs required to place the lining.
- Costs for structural repairs of the concrete.

- Replacement costs of the reinforcement.
- Costs due to indirect consequences of failure.
- Costs related to emptying and filling the WRS during crack repair.
- Inspection costs to assess the extent of repairs required.

The failure costs are assumed to be $C_f = R1500/m^2$. This is the cost to provide water tight lining per square meter of the WRS. This seems reasonable from a practical perspective.

4.2.2.3 Cost Ratio C_f/C_1

From the quantified costs, C_1 and C_f, the cost ratio can be calculated for flexural and tensile cracking. The cost ratios are calculated using the generic framework (C_f/C_1). The cost ratios for both WRS are shown in table 4.11. From a practical perspective, these cost ratios seem reasonable.

Table 4.11: Cost Ratio C_f/C_1 for the WRS

	C_1 $[R/m^2]$	C_f $[R/m^2]$	C_f/C_1
WRS 1 F-LT	256.62	1500.00	5.9
WRS 1 T-LT	240.58	1500.00	6.2
WRS 2 F-LT	344.30	1500.00	4.5
WRS 2 T-LT	240.21	1500.00	6.2

The first observation, made from table 4.11, is that the cost ratios for T-LT for WRS 1 and WRS 2 are the same. This means that $\beta_{t,SLS}$ is the same for both structures. On the other hand for F-LT, the cost ratio for WRS 1 is significantly higher than the cost ratio for WRS 2. This is because the cost of providing reinforcement for WRS 1 is cheaper (smaller bar diameter) than WRS 2. The second observation made from these results is that flexural cracking has a lower cost ratio than tensile cracking.

This is because flexural cracking has a larger A_{s_L} than tensile cracking.

4.2.3 β_{tSLS} for the WRS

As V_E and C_f/C_1 have been quantified, $\beta_{t,SLS}$ for the two WRS is obtained from table 3.3. In this paper the statistical parameters for the two WRS did not vary, resulting in the same V_E for both structures. The cost ratios were also quantified to be different for F-LT but the same for T-LT. Table 4.12 summarises the results of the previous sections. From table 3.3 the target values $\beta_{t,SLS}$ can be determined. The results from table 3.3 are also shown in table 4.12.

Table 4.12: $\beta_{t,SLS}$ for the WRS from the Generic Development

	WRS	V_E	C_f/C_1	$\beta_{t,SLS}$
F-LT	1	0.373	5.9	1.58
	2	0.373	4.5	1.48
T-LT	1	0.332	6.2	1.68
	2	0.332	6.2	1.68

4.3 Specific $\beta_{t,SLS}$ for the WRS Determined from Cost Optimisation

The assumption made in the generic development of chapter 3 that E has a LN distribution does not hold for the WRS. E for the crack width prediction of the WRS has a N distribution as a result of the N distribution of the model uncertainty. To check if the generic $\beta_{t,SLS}$ values still hold for the WRS with a normal distribution of E a full reliability analysis and cost optimisation is done. The results from the cost optimisation are then compared to the results obtained from table 3.3.

4.3.1 FORM Analysis

A FORM analysis in Risk Tools (RT) [41] on the limit state equation is performed to determine $T(A_s)$ of the WRS. E is replaced in the limit state equation with $E = \theta w_{m,max}$. As the crack width is calculated from the random variables of the input, the crack width equation is inserted into E rather than the predicted mean-maximum crack width. The input limit state equations are shown in equations 4.10 and 4.11 for flexure and tension respectively. The input variables and their assumed statistical parameters remain the same as for the Monte Carlo analysis and are shown in table 4.7.

$$g(1, w_{m,RT}(F)) = 0.2 - \theta[3.4c + 0.425k_1k_2(\frac{\phi h_c b}{A_s})] \times [\frac{M_s}{zA_sE_s} - \frac{k_t f_{ct} h_c}{A_s}\frac{1 + \alpha_e A_s/(h_c b)}{E_s}] \tag{4.10}$$

$$g(1, w_{m,RT}(T)) = 0.2 - \theta[3.4c + 0.425k_1k_2(\frac{\phi h_c b}{A_s})] \times [\frac{T_s}{A_sE_s} - \frac{k_t f_{ct} h_c}{A_s}\frac{1 + \alpha_e A_s/(h_c b)}{E_s}] \tag{4.11}$$

For this example, A_s (the decision parameter) varies in increments of $\Delta A_s = 100mm^2$ for the four different scenarios. For WRS 1 flexure and tension, the range $A_s = [2900 - 6000]mm^2$ is considered. For WRS 2 flexure and tension, the ranges $A_s = [4000 - 7500]mm^2$ and $A_s = [3000 - 5500]mm^2$ are considered. The ranges were chosen based on the range of the generic case such that $d = 0.2/E \approx 3$.

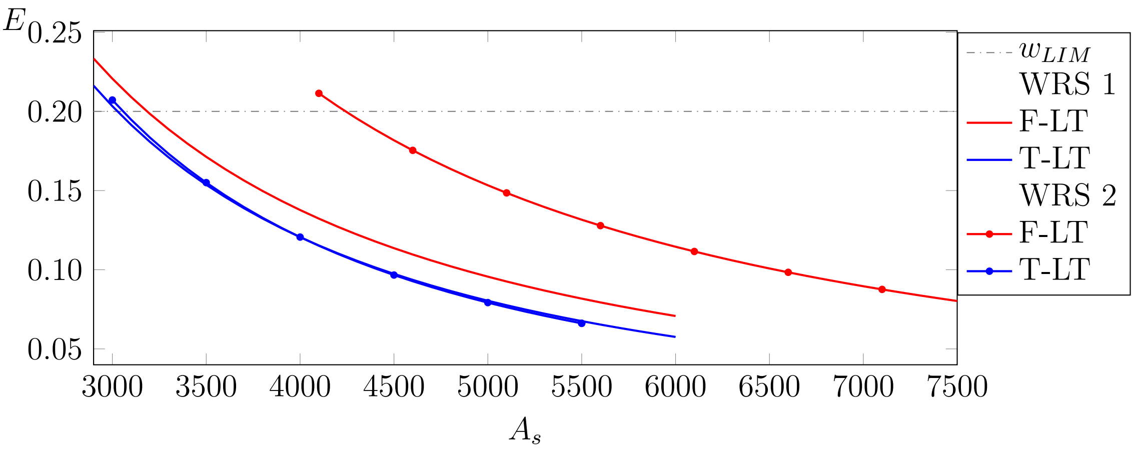


Figure 4.5: E for Varying A_s

E is shown in figure 4.5, the calculations are shown in appendix B. As was identified in section 4.1.3, E has the same amount of reinforcement for both WRS in tension. This is because changing the thickness of the section has no influence on the steel stress in tension. The thickness of the section does, however, have a significant role on the flexural prediction model resulting in more reinforcement required for a thinner section. This influence on E is further observed with the results from the FORM analysis.

Equations 4.10 and 4.11 are entered into RT and a FORM analysis is performed. θ and consequently E are considered with both a LN and N distribution. The $\beta(A_s)$ values obtained are shown in figure 4.6 as the $\beta(A_s)$ versus the amount of reinforcement A_s. The influence of the distribution choice is not negligible towards the tail end of E. The influence on the assumption of the distribution is further observed with the cost optimisation of the WRS.

As identified, h has a negligible influence on E for the tensile load case of the WRS. As expected there is also a negligible difference for $\beta(A_s)$ for tensile crack width predictions.

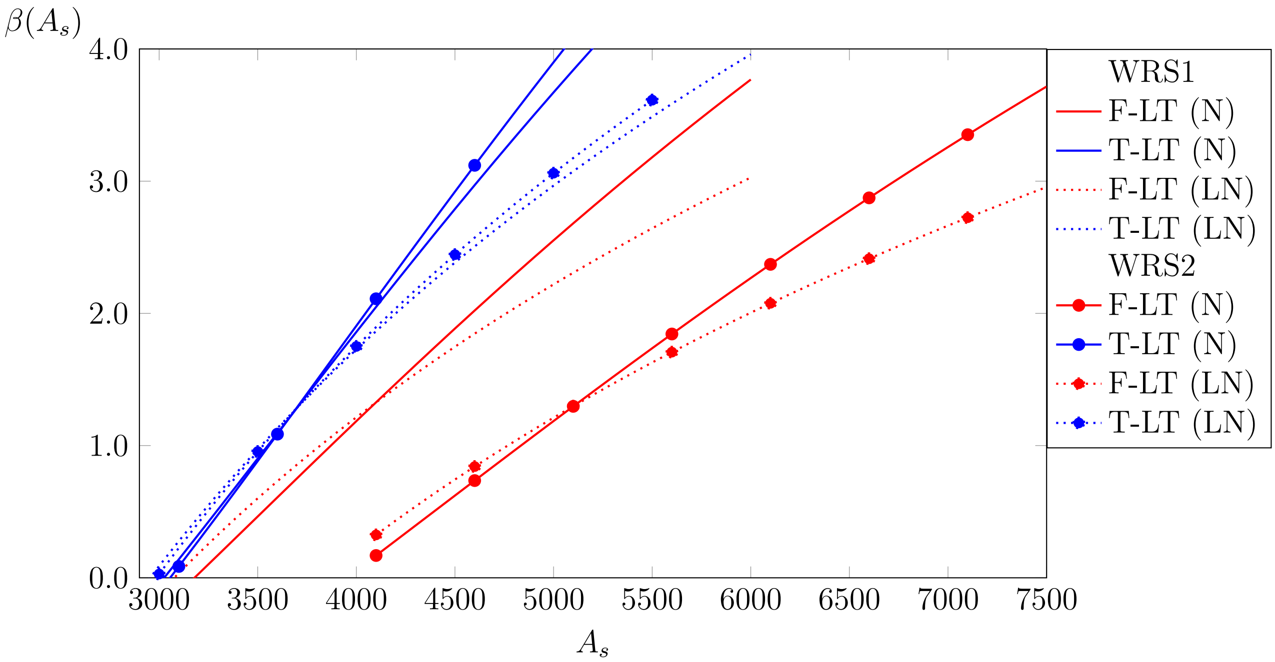


Figure 4.6: $\beta(A_s)$ vs A_s of WRS

4.3.2 $\beta_{t,SLS}$ from Cost Optimisation

For the WRS, A_s is chosen to be the decision parameter. To perform a cost optimisation the cost of providing safety C_1 depends on how many bars are required to provide sufficient reinforcement per meter. C_1 and C_f are determined using equations 4.13 and 4.14 respectively. Cost optimisation is then performed using equation 4.12 as described in section 3.1.

$$C_{tot} = C_1 A_s + C_f p_f(A_s) \tag{4.12}$$

Where:

$$C_1 = \frac{1}{A_{s1}} \times w_s \times 10.20 \tag{4.13}$$

$$C_f = R1500/m^2 \tag{4.14}$$

with	C_1	Cost of providing reinforcement per square meter per square millimeter increase in A_s for the WRS
	C_f	Failure costs for the WRS
	A_s	Amount of reinforcement - decision parameter for the WRS
	$p_f(A_s)$	Probability of failure as a function of A_s
	w_s	Weight of one bar of reinforcement

The results for the economic optimisation are shown in table 4.13. The difference between the $\beta_{t,SLS}$ for LN and N is negligible. The two WRS resulted in the similar $\beta_{t,SLS}$ values. This is expected as the statistical parameters for the two structures remained the same and the costs resulted in similar cost ratios for the two cases.

It was predicted that the results from the cost optimisation of the WRS would match the results of the $\beta_{t,SLS}$ values determined from table 3.3. However, this is not the case for the two structures. For the comparison of the two results, a difference of $\Delta\beta_{t,SLS} \approx 0.6$ and $\Delta\beta_{t,SLS} \approx 0.4$ is observed for a N and LN distribution of E respectively. From this difference in the results, it is evident that an assumption or generalisation made for the generic structure that does not hold for the WRS. The assumption that E has a LN distribution has no significant influence on the discrepancy of the results. An investigation into what caused this discrepancy is done in chapter 6.

Table 4.13: $\beta_{t,SLS}$ for the WRS

	WRS	From Table 3.3 Section 4.2.3	Cost Optimisation N	LN
F-LT	1	1.6	2.2	1.9
	2	1.5	2.1	1.9
T-LT	1	1.7	2.2	2.1
	2	1.7	2.3	2.2

4.3.3 Concluding Remarks

This chapter determined the $\beta_{t,SLS}$ for two WRS. First, E of the WRS is determined, by using the Eurocode's crack width prediction model $w_{m,max}$ multiplied by the model uncertainty θ based on the same prediction model. The two WRS differed in thickness, cover and bar diameter. The prediction of the crack width is considered for both flexural and tensile cracking as well as long and short term loading. The amount of reinforcement A_{s_L} is determined so that $E = L$ (where $L = w_{lim} = 0.2$). For both flexural and tensile cracking, A_{s_L} is governed by long term loading for the two structures investigated.

Once E is determined the coefficient of variation V_E could be determined. V_E is determined by both Monte Carlo Simulation and an approximation that assumes normally distributed uncorrelated variables. The approximation is identified as a suitable method for determining V_E as the results obtained are similar to the MCS. From the MSC, the distribution of E is determined to be N as a result of the N distribution of θ. The generic development of chapter 3 assumed E to be lognormally distributed, θ and consequently E are considered to have both N and LN distributions. V_E is calculated as 0.373 and 0.332 for flexure and tension respectively. The two WRS had the same distribution for E.

The next step was to quantify the costs for the specific structure. The cost of providing reinforcement C_1 per square meter was calculated for A_{s_L} (resulting in $\mu_E = w_{lim}$) and the failure costs were assumed to be $C_f = R1500/m^2$. For F-LT, the cost ratios were quantified as $C_f/C_1 = 5.9$ and 4.5 for WRS 1 and 2 respectively, and for T-LT, $C_f/C_1 = 6.2$ for both WRS.

$\beta_{t,SLS}$ was then read from table 3.3 and is summarised in table 4.13. To determine if these target values from the generic development hold for the specific application, both normal and lognormal distributions of E, a cost optimisation is done. These results are summarised in table 4.13. A discrepancy in the results obtained from the generic development and the specific structure occurred. The reason for this discrepancy is further investigated in the subsequent chapters.

5. Example 2: Simply Supported Beam

As an additional example, the target reliability for the deflection of a simply supported beam (SSB) is determined from the generic development (table 3.3). The target reliability $\beta_{t,SLS}$ is read from the table 3.3 and compared to the $\beta_{t,SLS}$ from the specific cost optimisation. This is done following a similar procedure for the WRS and is shown in figure 5.1.

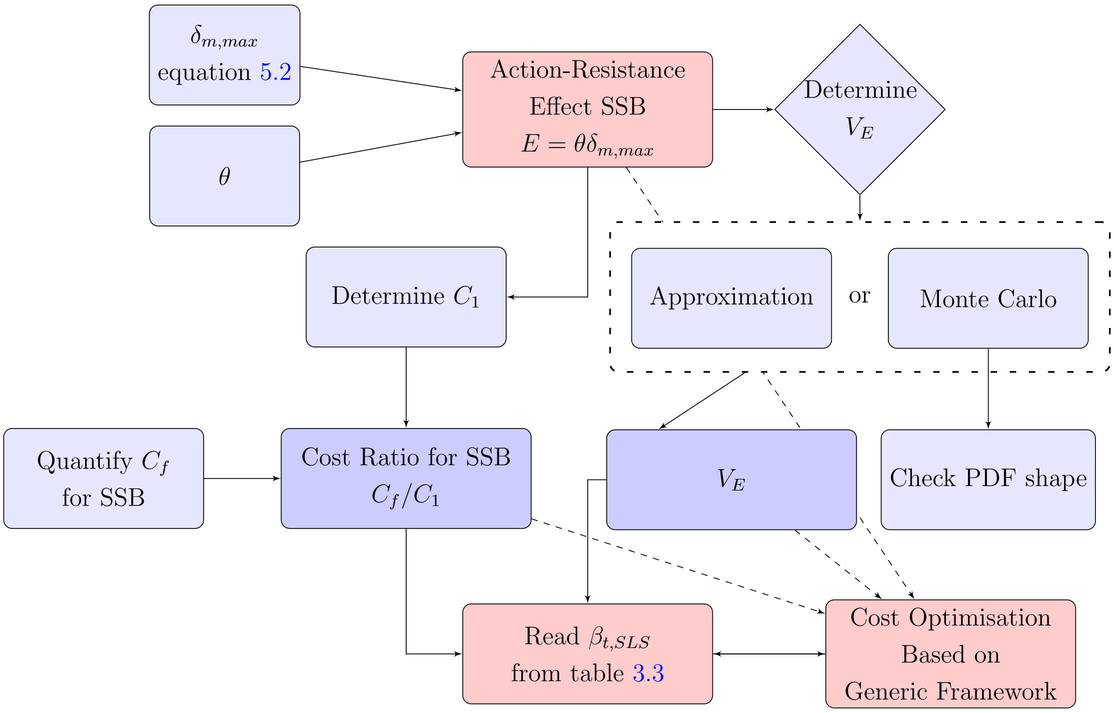


Figure 5.1: Framework - Determining $\beta_{t,SLS}$ for SSB

5.1 Action-Resistance Effect

From the generalised limit state equation (equation 3.7) the action-resistance effect is defined as $E = \theta Y$ (equation 3.8). The model uncertainty θ is taken to be $\mu_\theta = 1$ for this simple example. For deflections the predicted serviceability condition Y is the mean value prediction of the maximum deflection $\delta_{m,max}$ of the SSB. $\delta_{m,max}$ is based on the Eurocode prediction as discussed in section 5.1.1.

5.1.1 Deflection Equation

$\delta_{m,max}$ is calculated, following the provisions of section 7.4.3 in EN1992-1-1 [8]. Honfi's [21] interpretation of equation 5.1 is used as shown in equation 5.2. This is based on the well-known maximum deflection equation for a simply supported beam ($\frac{5}{384}\frac{qL^4}{EI}$).

$$\alpha = \xi\alpha_{II} + (1-\xi)\alpha_I \tag{5.1}$$

$$\delta_{m,max} = \xi\frac{5}{384}\frac{qL^4}{E_{c,eff}I_2} + (1-\xi)\frac{5}{384}\frac{qL^4}{E_{c,eff}I_1} \tag{5.2}$$

with	α	Deformation parameter
	α_I	Deformation - uncracked
	α_{II}	Deformation - fully cracked
	ξ	Distribution coefficient (allowing for tension stiffening)
	q	Distributed Load
	L	Span of the beam
	$E_{c,eff}$	Long term effective concrete modulus taking shrinkage into account
	I_1	Moment of inertia - uncracked
	I_2	Moment of inertia - fully cracked

The distribution coefficient ξ which allows for tension stiffening is calculated using equation 5.3. As the beam is subjected to a flexural load, ξ is written in terms of the SLS moment M_s and cracking moment M_{cr}. The distributed load q is a combination of the permanent and imposed load (G and Q) over the span of the beam. The long term effective concrete modulus is calculated using equation 5.4. The moment of inertia for the uncracked and cracked section is shown in equations 5.5 and 5.6 respectively. Equation 5.6 comes from course notes [51]. For this example it is assumed that the load which induces deflections is a sustained load, therefore, $\beta_\xi = 0.5$. The moments M_{cr} and M_s are calculated using equations 5.7 and 5.8 respectively.

$$\xi = 1 - \beta_\xi (\frac{M_{cr}}{M_s})^2 \tag{5.3}$$

$$E_{c,eff} = \frac{E_{cm}}{1 + \varphi(\infty, t_0)} \tag{5.4}$$

$$I_1 = \frac{bh^3}{12} \tag{5.5}$$

$$I_2 = \frac{bx^3}{12} + bx(\frac{x}{2})^2 + A_s(d^* - x^2) \tag{5.6}$$

$$M_{cr} = \frac{0.65\sqrt{f_{cm}} I_1}{h/2} \tag{5.7}$$

$$M_s = \frac{qL^2}{8} \tag{5.8}$$

with	β_ξ	Coefficient taking the influence of load duration into account
	M_{cr}	Cracking moment
	M_s	Maximum SLS moment in the beam
	E_{cm}	Concrete modulus
	$\varphi(\infty, t_0)$	Creep coefficient (determined from figure 3.1 in EN1992-1-1 [8])
	b	Width of the beam
	h	Height of the beam
	x	Depth to the neutral axis
	d^*	Depth to reinforcement
	A_s	Area of reinforcement
	f_{cm}	Mean value of the concrete compressive strength

5.1.2 Prediction of the SSB Action-Resistance Effect

The action-resistance effect of the SSB is determined for when $E = L$ (or in this case when $\theta\delta_{m,max} = \delta_{lim}$). The decision parameter for this example is chosen as a unit of h [42]. h is varied to determine where the deflection is equal to the limit δ_{lim} ($h = h_L$). The deflection limit according to EN1992-1-1 is $\delta_{lim} = L/250$ [8].

The sectional and material properties of the SSB are summarised in table 5.1. The amount of reinforcement is determined for ULS failure and is provided by 3Y32. The deflection of a rectangular simply supported beam is determined for a uniformly distributed load q. For the mean value prediction of the SLS deflection calculation the mean values of the permanent and variable loads are taken and not the design values. The mean value of the concrete compressive strength f_{cm} is taken from table 3.1 in EN1992-1-1 [8] and corresponds to $f_{cu} = 30MPa$.

Table 5.1: Simply Supported Beam - Assumed Variables

Variable	Symbol	Value	Units
Length	L	7.5	m
Width	b	350.0	mm
Thickness	h_L	485.0	mm
Cover	c	25.0	mm
Concrete Compressive Strength	f_{cm}	38.0	MPa
Concrete Modulus	E_c	31.0	GPa
Steel Yield Stress	f_y	450.0	MPa
Steel Modulus	E_s	200.0	GPa
Diameter	ϕ	32.0	mm
Area of reinforcement	A_s	2412.7	mm^2
Permanent Load	G	8.0	kN/m
Imposed/Variable Load	Q	5.0	kN/m

Some simplifications are made to simplify the limit state equation for the Monte Carlo Simulation and FORM analysis. First the long term effective concrete modulus $E_{c,eff}$ (equation 5.4) is assumed to be $13MPa$. This assumption comes from the assumption in chapter 4 that the modular ratio is $\alpha_E = 15$. The second assumption is to determine the fully cracked moment of inertia I_2 as a percentage f of the uncracked moment of inertia I_1. This is shown in equation 5.9. For this example $f = 37\%$ and is determined as the average percentage I_2 is of I_1 based on the investigated range of $h = [470 - 620]mm$. The calculations showing the difference in $\delta_{m,max}$ based on this assumption is shown in appendix C. As seen in figure 5.2 this simplification does not have a significant effect on $\delta_{m,max}$.

$$I_2 = f\frac{bh^3}{12} \tag{5.9}$$

Figure 5.2 shows the calculated μ_E (for $\mu_\theta = 1.0$) for varying h. The depth of the section is studied for the range $h = [470 - 620]mm$ in increments of $\Delta h = 5mm$. The deflection requirement ($\delta_{lim} = 30$) is satisfied when $h_L = 485mm$. Both the extensive calculation of $\delta_{m,max}$ using equation 5.6 to determine the cracked moment of inertia and the simplified calculation of $\delta_{m,max}$ using equation 5.9 to determine the cracked moment of inertia are shown in figure 5.2.

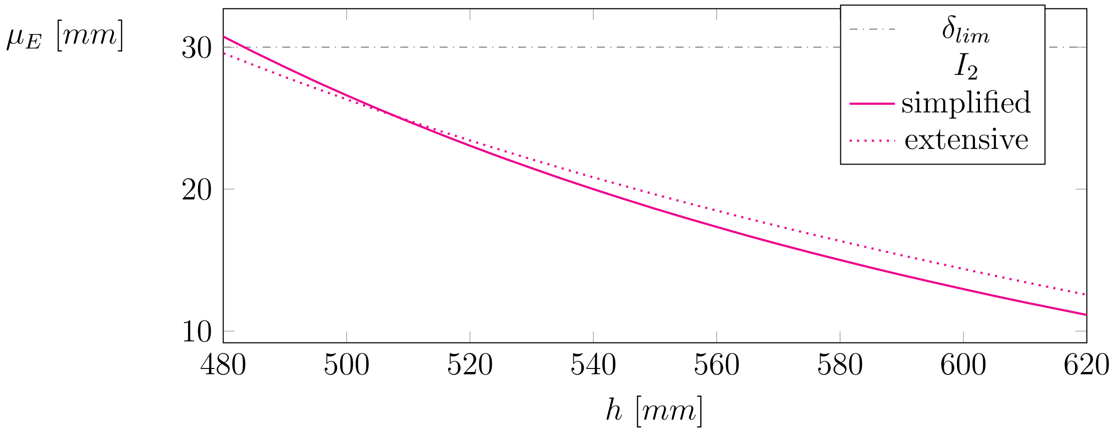


Figure 5.2: Deflection for Varying h

5.2 Generic $\beta_{t,SLS}$ for the SSB Obtained from the Generic Table

This section shows how to obtain the generic $\beta_{t,SLS}$ for the SSB from table 3.3. First the coefficient of variation of the SSB is computed. Next the cost ratio for the SSB is quantified. From these specific values for the SSB, $\beta_{t,SLS}$ can be obtained.

5.2.1 Calculation of V_E for the SSB

The coefficient of variation of the action-resistance effect V_E is necessary to obtain $\beta_{t,SLS}$ from table 3.3. Two methods are used to determine V_E. First, Monte Carlo Simulation (MCS) is used to accurately determine $V_{E_{MCS}}$ and the distribution of E. An approximation of $V_{E_{APP}}$ is done and compared to the results of the MCS. These two methods are described in section 4.2.1

The random variables of the deflection prediction model and the model uncertainty corresponding to this deflection prediction model are the input variables for the MCS. These random variables and their statistical properties are shown in table 5.2. The assumptions for the random variables are made based on what is currently available in literature. From table 5.2, the probability density function (PDF) of the variables is either deterministic (det), Normal (N), or lognormal (LN).

Table 5.2: Statistical Parameters of the SSB

Variable		**Units**	**PDF**	**Mean**	**V**	**Ref**
Dead Load	DL	kN/m	N	8	0.05	[18]
Live Load	LL	kN/m	LN	5	0.20	[18]
Length	L	m	det	15	0.00	[18, 26]
Width	b	m	det	350	0.00	[18, 26]
Beam Height	h_L	mm	LN	485	0.02	[18, 26]
Concrete Modulus	$E_{c,eff}$	GPa	LN	13	0.04	[21]
Concrete Compressive Strength	f_{cm}	MPa	LN	38	0.17	[21]
Model Uncertainty	θ		LN	1	0.10	[21]

The results from the MCS using @RISK are summarised with the results from the approximation in table 5.3. The distribution of E for SSB was fit using @RISK to a normal and lognormal distribution. This is shown in figure 5.3. It is observed in figure 5.3 that E has a LN distribution.

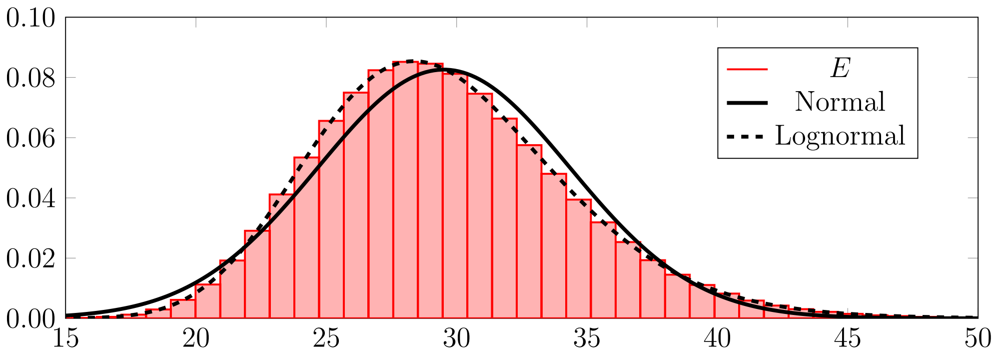


Figure 5.3: Histogram of the Action-Resistance Effect for the SSB

For the SSB, $V_{E_{APP}}$ is almost double $V_{E_{MCS}}$. $V_{E_{MCS}}$ is determined by sampling random variables whereas $V_{E_{APP}}$ assumes that the variables are uncorrelated and normally distributed, therefore, $V_{E_{MCS}}$ is correct for the SSB. The difference in V_E for the two methods is due to two reasons. The first is that the approximation is made on the assumption that normally distributed uncorrelated variables are used. This

Table 5.3: V_E for the SSB

	PDF	MCS	APP
		$V_{E_{MCS}}$	$V_{E_{APP}}$
Deflections	LN	0.164	0.287

is not true for deflections as most of the random variables have a LN distribution. The second reason is from the complexity of the deflection equation. For the WRS, V_θ was so large that it governed V_E. However, for the SSB, V_θ is relatively small compared to the other random variables such that it had a negligible influence on V_E.

Using the higher $V_{E_{APP}}$ results in a significantly lower $\beta_{t,SLS}$ than the $\beta_{t,SLS}$ obtained from the lower $V_{E_{MCS}}$. The approximation results in an unconservative recommendation for the target reliability of the SSB. It is, therefore, recommended that a full MCS or sampling analysis be performed to determine V_E.

5.2.2 Calculation of the Cost Ratio

To obtain $\beta_{t,SLS}$ from table 3.3, the specific cost ratios C_f/C_1 also need to be quantified. This is done by determining the specific costs of providing safety C_1 and the specific failure costs C_f. The costs in this section are a simple representation of some of the cost considerations when determining C_f/C_1. As a consistent unit of measurement is necessary, the costs for the SSB are calculated in Rands (R).

For the generic structure, the cost of providing safety is equal to the cost of one unit of d. Since $d = 1$ implies that $E = L$ for the generic structure (equation 3.9), the cost of providing concrete C_1 for the SSB must correlate to $\mu_E = \delta_{lim}$. For the SSB, the decision parameter is the depth of the section h. Therefore C_1 is the cost of providing h_L such that $\mu_E = \delta_{lim}$. h_L is the depth of the beam such that $\mu_E = \delta_{lim}$. It is assumed that all other construction costs C_0 are independent of h_L.

To determine C_1 for the SSB, the cost to provide concrete. The cost of concrete per cubic meter is $R2500/m^3$ [29]. C_1 is then determined using equation 5.10 as the cost per per cubic meter(R/m^3) times the width (m) times the height (m) times length

(L). The cost C_1 is calculated as $C_1 = R3182.85$.

$$C_1 = bh_L L \times R2500/m^3 \tag{5.10}$$

with b Width ($b = 0.350m$)
h_L Height so that $\mu_E = \delta_{lim}$ ($h = 0.485m$)
L Length ($L = 7.5m$)

For the SSB, the failure costs C_f are simply assumed to be equal to $R6000$ and is the cost associated with strengthening the beam. From the quantified costs, C_1 and C_f, the cost ratio can be for the SSB. The cost ratios are calculated $C_f/C_1 = 1.9$. This cost ratio seems reasonable from a practical perspective.

5.2.3 $\beta_{t,SLS}$ for the SSB

As V_E and C_f/C_1 have been quantified, $\beta_{t,SLS}$ for the SSB is read from table 3.3. V_E is taken as the result of the MCS and the approximation (APP). This is to show how an error in calculating V_E influences the resulting $\beta_{t,SLS}$.

Table 5.4: $\beta_{t,SLS}$ for the SSB from the Generic Development

	V_E	C_f/C_1	$\beta_{t,SLS}$
MCS	0.157	1.9	1.61
APP	0.287	1.9	1.16

Table 5.4 highlights where $\beta_{t,SLS}$ for the SSB is read from the table. The error in the V_E from the approximation is apparent. For the V_E from MCS, $\beta_{t,SLS} = 1.6$ while for the approximation $\beta_{t,SLS} = 1.2$. This results in $\Delta\beta_{t,SLS} = 0.4$ which is a considerably large difference in the recommended target reliability for the SSB.

5.3 Specific $\beta_{t,SLS}$ for the SSB Determined from Cost Optimisation

To determine if the discrepancy in the generic development of $\beta_{t,SLS}$ also occurs for deflections, it is necessary to perform cost optimisation for the SSB. Cost optimisation is performed on μ_E for the SSB. As increasing A_s has a negligible effect on the deflection, it is not a suitable choice for the decision parameter for deflections. On the other hand, small increments in the section's depth h have a significant influence on the deflection, thus making it a suitable choice for the decision parameter.

5.3.1 FORM Analysis

A FORM analysis is done using Risk Tools (RT) [41] on the limit state equation to determine $\beta(h)$ of the SSB. E is replaced in the limit state equation with $E = \theta\delta_{m,max}$. The input limit state equation for deflection is shown in equation 5.11, which is determined based on the assumption in equation 5.9 and simplified as shown in appendix C.

$$g(\delta_{lim}, \delta_{m,maxRT}) = 30 - \theta\frac{5(DL+LL)L^4}{32E_{c,eff}bh^3} \left[1 + (\frac{1}{f} - 1)(1 - \frac{169\beta_{\xi}f_{cm}}{225}(\frac{bh^2}{(DL+LL)L^2})^2)\right] \tag{5.11}$$

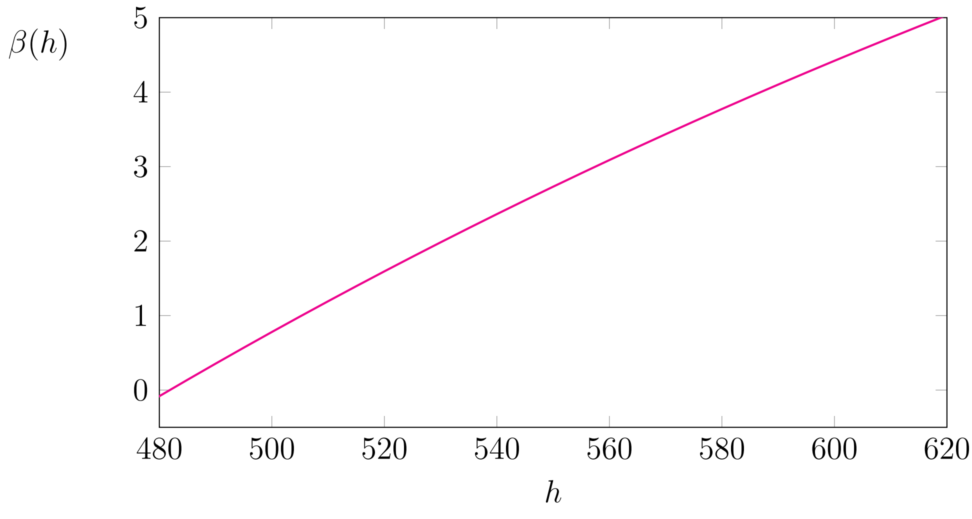


Figure 5.4: $\beta(h)$ vs h of the SSB

The input variables and their assumed statistical parameters are the same as for the MCS as shown in table 5.2. For this example h (the decision parameter) is studied for the range $h = [470 - 620]mm$ in increments of $\Delta h = 5mm$. The $\beta(h)$ values obtained from the FORM analysis are shown in figure 5.4 as the $\beta(h)$ versus the depth of the section h.

5.3.2 $\beta_{t,SLS}$ from Cost Optimisation

The costs in this section are a simple representation of some of the cost considerations when determining C_f/C_1. As a consistent unit of measurement is necessary, the costs for the SSB are calculated in Rands (R).

For the SSB, h is chosen to be the decision parameter. The cost of providing concrete C_1 depends on how much concrete is required per meter of h. The cost of concrete per cubic meter is $R2500/m^3$ [29]. C_1 is then calculated using equation 5.13. The failure costs C_f are assumed to be the cost of strengthening the beam and are equal to $R6000/m$ (per meter of beam).

Once the costs have been quantified cost optimisation is then performed using equation 5.12 as described in section 3.1.

$$C_{tot} = C_1 h + C_f p_f(h) \tag{5.12}$$

Where:

$$C_1 = bL \times R2500/m^3 = R6562.50/m \tag{5.13}$$

with	C_1	Cost of providing concrete per meter increase in h for the SSB
	C_f	Failure costs for the SSB
	h	Height - decision parameter for the SSB
	$p_f(h)$	Probability of failure as a function of h
	b	Width ($b = 0.350m$)
	L	Length ($L = 7.5m$)

From cost optimisation $\beta_{t,SLS} = 2.4$. Compared to the result read from table 3.3 ($\beta_{t,SLS} = 1.6$), a discrepancy of $\Delta\beta_{t,SLS} = 0.8$ occurs. This is significantly larger than the discrepancy for the WRS. The discrepancy in both examples is discussed in chapter 6.

5.4 Concluding Remarks

This chapter determined the $\beta_{t,SLS}$ for a SSB. First, E of the SSB is determined. This is done by using the mean value prediction of the Eurocode's maximum deflection prediction model $\delta_{m,max}\ (Y)$ and the model uncertainty θ based on the same prediction model. The height of the beam h is determined so that $E = L$ (where $L = \delta_{lim} = 30mm$).

Once E is determined the coefficient of variation V_E could be determined. V_E was determined by both Monte Carlo Simulation and an approximation. The approximation is not a suitable method for determining V_E as the results obtained are almost double the results from the Monte Carlo Simulation. This error in the V_E obtained from the approximation is due to the assumption of normally uncorrelated variables being violated by the lognormally distributed random variables of $\delta_{m,max}$. From the Monte Carlo analysis, the distribution of E is determined to be lognormal.

The results from the generic development and specific cost optimisation are shown in table 5.5. The discrepancy in the results is further discussed in the next chapter.

Table 5.5: $\beta_{t,SLS}$ for the SSB

	From Table 3.3	Cost Optimisation
SSB	1.6	2.4

6. Discussion of the Discrepancy

A discrepancy in the $\beta_{t,SLS}$ obtained from the generic development (table 3.3) and the $\beta_{t,SLS}$ determined from cost optimisation occurs for both the water retaining structure, WRS, and simply supported beam, SSB. As this occurs for both crack widths (WRS) and deflections (SSB), the discrepancy lies within the generic simplifications for determining $\beta_{t,SLS}$ and not the specific cost optimisation of the examples. In other words, it is assumed that the specific $\beta_{t,SLS}$ obtained from cost optimisation of the specific structure is correct.

In this chapter, "generic" refers to the generic limit state equation, decision parameter d, and $\beta_{t,SLS}$ from table 3.3 while "specific" refers to the specific limit state equation of the examples, the decision parameters A_s and h, or the $\beta_{t,SLS}$ obtained from cost optimisation of the example structure considered.

6.1 Investigation into the Cost Equation

The discrepancy between the generic $\beta_{t,SLS}$ and the specific $\beta_{t,SLS}$ is determined by an investigation of the generic cost equation. The total costs of the structure which are to be minimized is shown in equation 6.1. Equation 6.1 and its components are discussed in chapter 3 (equation 3.1).

$$C_{tot} = C_0 + C_1 d + C_f p_f(d) \tag{6.1}$$

with	d	Decision parameter(s)
	C_0	Initial costs independent of d
	C_1	Costs of providing safety; dependent on d
	C_f	Failure costs
	$p_f(d)$	Probability of failure as a function of d
	$C_0 + C_1 d$	Construction costs
	$C_f p_f(d)$	Expected failure costs

Deriving equation 6.1 with respect to d and setting it equal to zero results in equation 6.2. When the derivative of C_{tot} is equal to zero, the costs are at a minimum and the corresponding d is the optimum decision parameter. This is the foundation for the principles of cost optimisation. This is shown in figure 6.1.

$$\begin{aligned} \frac{\partial C_{tot}}{\partial d} &= C_1 + C_f \frac{\partial p_f(d)}{\partial d} = 0 \\ 1 + \frac{C_f}{C_1} \frac{\partial p_f(d)}{\partial d} &= 0 \end{aligned} \tag{6.2}$$

From equation 6.2, it can be seen that the optimum decision parameter d_{opt} and corresponding $\beta_{t,SLS}$ are dependent on three components: the cost of providing safety C_1 per unit of d, the failure costs C_f, and the partial derivative $\partial p_f(d)/\partial d$ of the probability of failure $p_f(d)$ with respect to d. Changing any of these components results in a shift in the d_{opt} value.

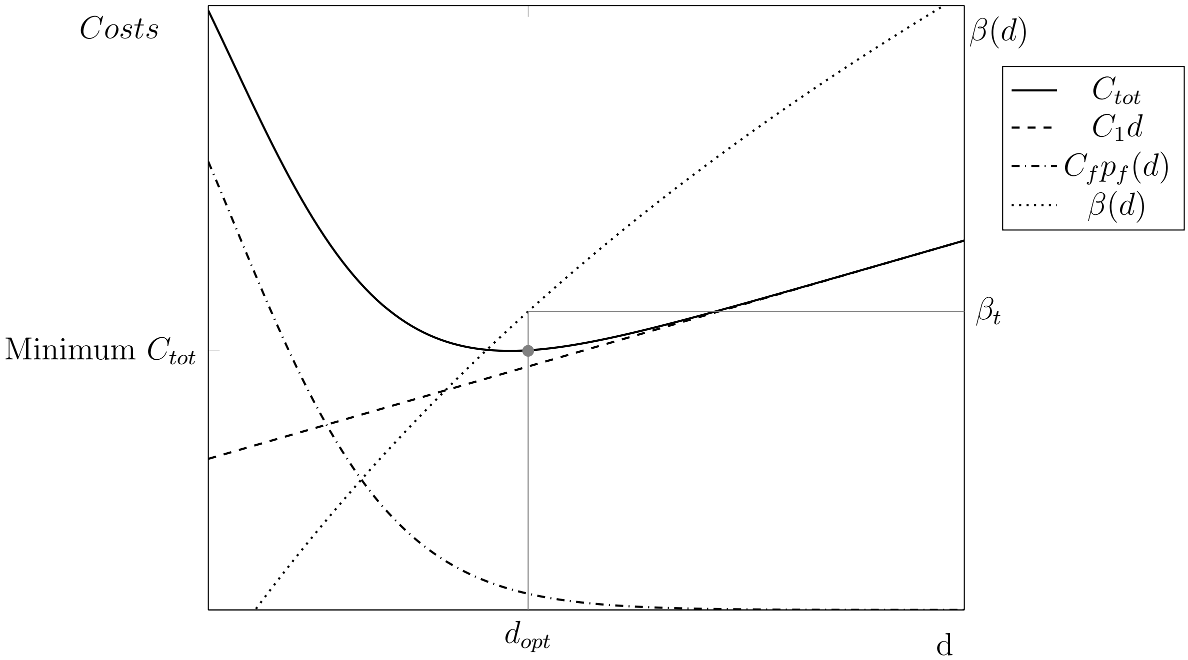


Figure 6.1: Concept of Cost Optimisation

The costs C_1 influence the position of d_{opt} based on the cost to increase d. Larger values of C_1 (expensive to increase d) imply that a safer structure is not economically viable and therefore has a lower d_{opt} and corresponding $\beta_{t,SLS}$. The failure costs C_f, on the other hand, imply the opposite. Large values of C_f involve expensive costs if failure does occur resulting in a larger d_{opt} and $\beta_{t,SLS}$. In other words, it is more economically viable to provide more safety.

The failure costs are also dependent on how efficient d is in decreasing the $p_f(d)$. For an inefficient d, it is not economically viable to provide a safer structure as the $p_f(d)$ does not change with increasing d, implying additional failure costs with little effect.

6.1.1 The Costs (C_1 and C_f)

For the generic development, the cost of providing safety and the failure costs are considered as a ratio. In the specific examples, C_f is quantified and C_1 is determined as the cost of providing the specific decision parameter. The total costs for the WRS and SSB are shown in equations 6.3 and 6.4 respectively. For the WRS, the amount of reinforcement A_s is the specific decision parameter and for the SSB, the height of the beam h is the specific decision parameter. To determine the specific cost ratio (such that the generic $\beta_{t,SLS}$ can be obtained from table 3.3), a relationship between the specific costs of providing reinforcement or concrete with the generic costs of providing safety is necessary.

$$C_{tot} = C_0 + C_1 A_s + C_f p_f(A_s) \tag{6.3}$$

$$C_{tot} = C_0 + C_1 h + C_f p_f(h) \tag{6.4}$$

To map the specific costs of the examples to the generic cost ratio, the relationship between the generic d and the specific decision parameters is considered. Since $d = 1$ implies that $E = L$, the amount of reinforcement A_{s_L} such that $\theta w_{m,max} = w_{lim}$ or height of the beam h_L such that $\theta \delta_{m,max} = \delta_{lim}$ is determined. The cost C_1 associated with providing this much reinforcement or concrete maps the specific C_1 to the generic.

From these costs, the generic cost ratio can be determined and the genric $\beta_{t,SLS}$ can be obtained for the specific examples. Based on how the costs are considered in the specific examples, it appears that the discrepancy is not because of C_1 or C_f. This implies that the discrepancy is lies with in the $\frac{\partial p_f(d)}{\partial d}$ component of the optimum solution.

6.1.2 The Decision Parameter and the Probability of Failure $p_f(d)$

The $\partial p_f(d)/\partial d$ component depends on the efficiency of d to decrease the $p_f(d)$. For the examples, the generic $\beta_{t,SLS}$ obtained from table 3.3 is lower than the specific $\beta_{t,SLS}$ determined from cost optimisation. This implies that the generic d does not decrease $p_f(d)$ as efficiently as the specific h decreases $p_f(h)$ or A_s decreases $p_f(A_s)$.

The decision parameter is linked to $p_f(d)$ through the action-resistance effect E of the limit state equation. For the generic development, E is defined to be the inverse of the decision parameter (for $L = 1$) as shown in equation 6.5. For both the WRS and the SSB, there is not an explicit relationship between E and the chosen decision parameter as shown in equations 6.6 and 6.7. In the specific cases, E is defined as a function of the chosen decision parameter.

$$\text{Generic:} E = \frac{1}{d} \tag{6.5}$$

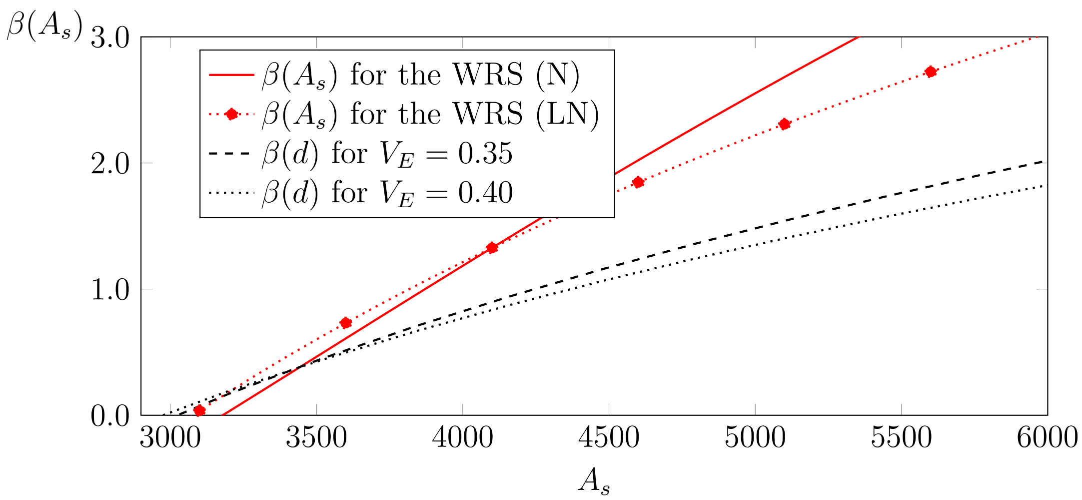
$$\text{WRS:} E = \theta w_{m,max}(A_s) \tag{6.6}$$

$$\text{SSB:} E = \theta \delta_{m,max}(h) \tag{6.7}$$

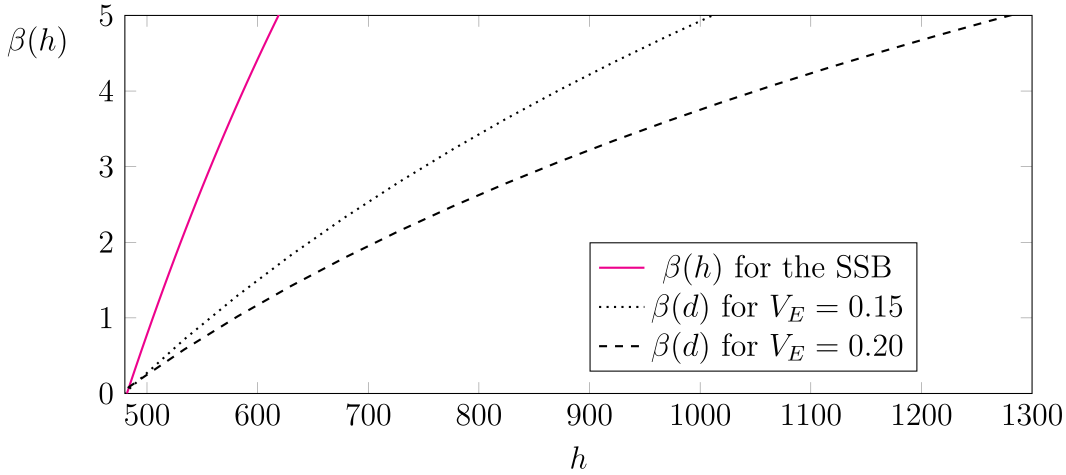
with

E	Action-resistance effect
d	Decision parameter of the generic structure
A_s	Decision parameter of the WRS
h	Decision parameter of the SSB
θ	Model uncertainty
$w_{m,max}(A_s)$	Mean-maximum crack width as a function of A_s
$\delta_{m,max}(h)$	Mean-maximum deflection as a function of h

The decision parameter plays a role in how quickly C_1 increases and how efficiently $p_f(d)$ decreases. For the specific examples, the specific decision parameters were determined such that $E = L$ and $d = 1$. However, a linear relationship between d and the specific decision parameters does not exist. Although $d = 1$ ($E = L$) implies that $A_s = A_{s_L}$ or $h = h_L$, $d = 2$ ($E = 0.5L$) does not imply that $A_s = 2A_{s_L}$ or $h = 2h_L$ for the WRS and SSB respectively. This indicates that it is more efficient to increase A_s or h than what is implied by d. This is shown in figure 6.2.



(a) WRS F-LT: Efficiency of A_s on $\beta(A_s)$ for $V_E = 0.373$



(b) SSB: Efficiency of h on $\beta(h)$ for $V_E = 0.164$

Figure 6.2: Efficiency of the Specific Decision Parameter on the Specific $\beta(d)$ Compared to the Implied Efficiency of d

For the WRS only $\beta(A_s)$ associated with flexural long term crack widths (F-LT) is shown as it is similar for the other cases. Figure 6.2a clearly shows that A_s is more efficient in increasing safety whereas d is less efficient for the WRS F-LT. This is also evident from equations 4.5 and 4.6 where the mean-maximum crack width equation includes $1/A_s^2$ for both flexure and tension. This implies that A_s is two times more efficient than d in decreasing the probability of failure. Similarly, figure 6.2b clearly shows that h is more efficient in increasing safety whereas d is less efficient in terms of the SSB. This is also evident from equation 5.1 where the mean-maximum deflection equation includes $1/h^3$. This implies that h is three times more efficient than d in decreasing the probability of failure.

Due to the resulting inefficiency of the generic d on $p_f(d)$, a lower $\beta_{t,SLS}$ is determined from the generic development than what is determined for the specific structure. This is because a less efficient increase in the reliability of the structure results in a lower optimum decision parameter as it is not economical to provide a higher level of reliability. Although the assumption of $E = 1/d$ is a reasonable assumption to account for a range of structures, it is unconservative as it results in lower $\beta_{t,SLS}$ values than what the reality of the specific examples suggest.

6.2 Proposed Solution

As the decision parameter does not account for the efficiency of the specific decision parameters in decreasing $p_f(d)$, this discrepancy in $\beta_{t,SLS}$ needs to be accounted for.

The first solution would be to define a more efficient d in the generic development of E. However, it may not be possible to sufficiently define d such that it takes into account all serviceability prediction models, for example: $E = 1/d^2$ may account for the crack width prediction model but still be inefficient when compared to the deflection model. On the other hand, the efficiency of the decision parameter could be increased by adding some factor, for example: $E = 1/d + 3$. Further investigations, into the decision parameters of specific examples and how efficiently the decrease $p_f(d)$, are required for a range of serviceability prediction models (crack widths, deflections, stresses, vibrations). Defining a more efficient d requires further investigations and is out of the scope of this work.

The second solution would be to determine some efficiency parameter ϑ that takes into account the discrepancy in the efficiency of $p_f(d)$ in the generic sense. This efficiency parameter could be determined as a ratio of the partial derivative of the optimum solution of the generic d to the partial derivative of the specific solution. This is shown in equation 6.9.

$$1 + \frac{C_f}{C_1}\frac{\partial p_f(d_G)}{\partial d_G} = 1 + \frac{C_f}{C_1}\frac{\partial p_f(d_S)}{\partial d_S} \tag{6.8}$$

$$\frac{\partial p_f(d_G)}{\partial d_G} = \vartheta \frac{\partial p_f(d_S)}{\partial d_S} \tag{6.9}$$

with $\frac{C_f}{C_1}$ Cost ratio based on the generic framework
d_G Decision parameter of the generic structure
d_S Decision parameter of the specific structure (for example A_s or h)
ϑ Efficiency parameter

If an efficiency parameter can be defined such that it takes into account the difference in efficiency of the generic d_G compared to the specific d_S, the cost ratio of the specific strucutre can be mapped to the cost ratio of the generic framework, as shown in equation 6.10. The specific cost ratio is determined as discussed in the examples (chapters 4 and 5).

$$\frac{C_f}{C_1} = \vartheta \frac{C_f}{C_1} \tag{6.10}$$

with $\frac{C_f}{C_1}$ Cost ratio based on the generic framework
ϑ Efficiency parameter

7. Conclusion

The current recommendation in ISO2394 for the target reliability of the serviceability limit state is $\beta_{t,SLS} = 1.5$. However, it is unclear how this $\beta_{t,SLS}$ was determined. Specifically, for structures where the design is governed by the serviceability limit state (SLS), the target values should be consistent with cost-optimal principles. The goal of this project was to develop a generic framework to allow for easy estimation of reliability-based cost-optimal $\beta_{t,SLS}$ values for different applications.

7.1 The Generic Framework

The target reliability values, for the serviceability limit state based on a generic framework, were determined. The generic framework was established to perform economic optimisation to establish $\beta_{t,SLS}$ from the same principles that $\beta_{t,ULS}$ were calibrated. Cost optimisation was used to calculate the optimal $\beta_{t,SLS}$ for a range of failure consequence classes, cost of safety measures, and variability characterising different structures.

The principle of cost optimisation is to determine a reliability level that would minimize the total costs of the structure. The total costs of a structure comprise of the construction costs $C_0 + C_1 d$ plus the expected failure costs $C_f p_f(d)$. The cost C_1 is the cost of providing safety per unit of the decision parameter d. The failure costs C_f are the costs incurred if there is serviceability failure. From the formulation of the total costs, three components necessary for cost optimisation were identified: the costs C_f and C_1, the decision parameter d, and the reliability index $\beta(d)$ as a function of d.

A range of cost ratios C_f/C_1 was identified as the best method to generically account for the costs of the structure. Cost ratios from 0.5 up to 100 were considered. This range was chosen to take into account the influences of the costs and consequences associated with serviceability failure (failure costs C_f) compared to costs of increasing safety (costs per unit of d - C_1).

To determine $\beta(d)$, the first order reliability method (FORM) was identified as the best technique to perform a reliability analysis. To conduct a FORM analysis the limit state must be expressed as $g = 0$. From the limit state equation recommended in the Eurocode, the variables were quantified in the generic sense. The design limit L was assumed to be deterministic and the action-resistance effect E was identified as a function of random variables. E was defined by the product of the mean value prediction of the serviceability condition Y and the model uncertainty θ associated with the same prediction model.

For the generic structure it was determined that assigning E as random variable would be sufficient for the reliability analysis. The range for the mean values of the action-resistance effect μ_E was determined based on the relationship established between μ_E and the decision parameter d. From the generic limit state equation, μ_E is inversely proportional to d. The coefficient of variation V_E was varied parametrically to account for a wide range of structures. The range $V_E = [0.05 - 0.50]$ was chosen to account for the low variability of deflections and stresses to the high variability of cracking. Once this range was established, a reliability analysis was performed using FORM to determine the $\beta(d)$ values.

7.1.1 Conclusion of $\beta_{t,SLS}$ for the Generic Structure

From the range of cost ratios and $\beta(d)$ values for the range of V_E, economic optimisation was conducted to determine a parametric table of $\beta_{t,SLS}$ values. The $\beta_{t,SLS}$ values obtained are shown in table 7.1.

Table 7.1 covered a large range of structures for a range of serviceability conditions in a generic sense. It is expected that crack widths are covered by the range of $V_E = [0.15 - 0.45]$, depending on the degree of uncertainty associated with the model factor, and anticipated that for deflections and stresses the range $V_E = [0.05 - 0.25]$ would represent the $\beta_{t,SLS}$ for these structures. From this the necessity of such a large range for V_E is clear and reducing the range would result in some structures not being suitably represented. For a specific structure, $\beta_{t,SLS}$ can be determined based on the costs of increasing safety and the cost of failure for the specific structure combined with the variability of the action-resistance effect.

Table 7.1: $\beta_{t,SLS}$ for a One Year Reference Period

	C_f/C_1									
V_E	**0.5**	**0.8**	**1.0**	**1.5**	**2.0**	**4.0**	**10**	**20**	**50**	**100**
0.50	0.0	0.0	0.0	0.5	0.7	1.2	1.7	2.0	2.3	2.6
0.45	0.0	0.0	0.0	0.6	0.8	1.3	1.8	2.1	2.4	2.6
0.40	-0.1	-0.1	0.2	0.7	0.9	1.4	1.8	2.1	2.5	2.7
0.35	-0.1	0.0	0.4	0.8	1.1	1.5	1.9	2.2	2.6	2.8
0.30	-0.2	0.3	0.6	1.0	1.2	1.6	2.0	2.3	2.6	2.9
0.25	-0.3	0.5	0.8	1.1	1.3	1.7	2.1	2.4	2.7	3.0
0.20	0.1	0.8	1.0	1.3	1.5	1.9	2.3	2.5	2.8	3.1
0.15	0.7	1.1	1.3	1.5	1.7	2.0	2.4	2.7	3.0	3.2
0.10	1.1	1.5	1.5	1.8	2.0	2.3	2.6	2.8	3.1	3.4
0.05	1.6	1.9	1.9	2.1	2.3	2.6	3.0	3.2	3.3	3.7

7.2 The Applications

A study into the application of table 7.1 was conducted for three structures. Two water retaining structures (WRS), for the prediction of the crack width, and one simply supported beam (SSB), for the prediction of the maximum deflection, were investigated. $\beta_{t,SLS}$ was obtained from table 7.1 and calculated through cost optimisation for the three structures. The target values from the two methods were compared to ensure that the results were approximately equivalent.

E for the specific structures was calculated. For the WRS the amount of reinforcement A_{s_L} was determined so that $\mu_E = L$ (where $L = w_{lim} = 0.2mm$). The amount of reinforcement required A_s is governed by long term flexural and tensile loading for the two investigated structures. For the SSB, the beam's height h_L was determined so that $\mu_E = L$ (where $L = \delta_{lim} = 30mm$).

The coefficient of variation V_E was determined by both Monte Carlo Simulation (MCS) and a mathematical approximation that assumes the input variables are normally distributed. The approximation resulted in a similar V_E to the V_E obtained from the MCS for the WRS. However, for the SSB, the V_E from the approximation was almost double the V_E from the MCS. This was due to the lognormal distribution of the random variables of the action-resistance effect violating the assumptions on which the approximation is based.

From the MCS, the distribution of E was determined. For the WRS, θ was investigated as normal (N) and lognormal (LN). This resulted in corresponding N and LN distributions of E. A LN distribution was fit to E of the SSB. V_E was calculated as 0.373 and 0.332 for flexure and tension of the WRS respectively. For the SSB a smaller $V_E = 0.164$ was calculated.

The next step was to quantify the costs for the specific structures. The failure costs C_f were assumed to be the cost of providing liquid-tight lining for the WRS ($C_f = R1500/m^2$) and the cost to strengthen the beam for the SSB ($C_f = R6000$). It was assumed that there are no indirect failure costs for the two examples. The cost of providing safety C_1 is dependent on d in the generic framework. Since $d = 1$ implies $E = L$, the cost C_1 is calculated to be the cost of providing A_{s_L} for the WRS and h_L for the SSB. The cost ratios were quantified as $C_f/C_1 = 5.9$ and 4.5 for WRS 1 and WRS 2 flexural cracking respectively. For tension, $C_f/C_1 = 6.2$ for both WRS 1 and 2 as $A_{s_L} = 3000mm^2$ for both structures. For the SSB the cost ratio was quantified as $C_f/C_1 = 1.9$.

$\beta_{t,SLS}$ for the specific examples was then obtained from the generic framework (table 7.1). These target values $\beta_{t,SLS}$ are compared to the $\beta_{t,SLS}$ obtained from cost optimisation of the specific structures. The results from the cost optimisation did not consolidate with $\beta_{t,SLS}$ values obtained from the generic framework for both the WRS and SSB. The results from the applications are shown in table 7.2.

Table 7.2: Summary of $\beta_{t,SLS}$ for the Applications

			Cost Optimisation	
		Table 7.1	Normal	Lognormal
F-LT	WRS 1	1.6	2.2	1.9
	WRS 2	1.5	2.1	1.9
T-LT	WRS 1	1.7	2.2	2.1
	WRS2	1.7	2.3	2.2
SSB		1.6	-	2.4

7.2.1 Conclusion of the Generic $\beta_{t,SLS}$ based on the Applications

As observed in table 7.2, it is evident that there is a discrepancy between the generic framework for $\beta_{t,SLS}$ and the specific $\beta_{t,SLS}$ obtained from cost optimisation of the examples. The $\beta_{t,SLS}$ read from table 7.1 underestimate the $\beta_{t,SLS}$ of the specific structures.

It was anticipated that the assumption of E, having a lognormal distribution for the generic framework, would not always hold for a specific structure. For a structure where E has a non-lognormal distribution, the generic $\beta_{t,SLS}$ may not sufficiently account for the reliability of the structure. The WRS was investigated to have both a normal and lognormal distribution for both flexural and tensile cracking. For flexure (F-LT) the difference in the results from cost optimisation for a lognormally and normally distributed E is approximately $\Delta\beta_{t,SLS} \approx 0.2$. Tension (T-LT) on the other hand has a negligible difference of $\Delta\beta_{t,SLS} \approx 0.1$. However, from the results shown in table 7.2 it is evident that the distribution of E is not the reason for the discrepancy in the results.

The coefficient of variation V_E of the action-resistance effect of the specific structures is sufficiently covered by the range of V_E considered in the generic development. This implies that the discrepancy lies within the cost optimisation component of the generic framework. The partial derivative of the total costs with respect to d was investigated because when equal to zero, the optimum occurs. Three components were then identified to have an influence on the optimum; C_1, C_f and the partial derivative $\partial p_f(d)/\partial d$ of the probability of failure $p_f(d)$ with respect to d.

The first and second components, C_1 and C_f, were accounted for in the generic framework through the cost ratio C_f/C_1. For the specific examples, the costs of providing one unit of d was determined by calculating the cost associated with providing reinforcement or concrete such that $E = L$. It was concluded that the discrepancy was not a result of the cost components.

The third component which influences the position of the optimum is $\partial p_f(d)/\partial d$. The position of the optimum depends on the efficiency of d to decrease $p_f(d)$. It was identified that there was non-linear relationship between d of the generic framework and the specific decision parameters (A_s or h). Investigation into this non-linear relationship revealed that it was more efficient to increase A_s or h than what was implied by d. This resulted in higher target values for the examples from cost optimisation of the specific structures than what was determined from the generic framework.

The generic model is unconservative in the recommendations of $\beta_{t,SLS}$. However, the generic development set out by this thesis provides a suitable framework for determining $\beta_{t,SLS}$. As the generic model oversimplifies the SLS, further investigations into improving the generic framework should be conducted.

7.3 Recommendations

The generic framework and corresponding $\beta_{t,SLS}$ values provide a reasonable foundation for the reliability assessment of concrete structures governed by the SLS. This section lists the recommendations for further research into the target reliability related with SLS design.

This thesis identified a discrepancy in the the target values obtained from the generic framework and the cost optimisation of the specific example structures. The discrepancy is due to the lower efficiency of the decision parameter to decrease the probability of failure $p_f(d)$ in the generic case compared to the efficiency of decision parameters to do the same in the specific examples. Further investigations into increasing the efficiency of the generic decision parameter is necessary. However, it may not be possible to determine a generic decision parameter which accounts for different efficiencies of specific decision parameters of various SLS criteria. If this is the case, it may be necessary to determine an efficiency parameter to account for the discrepancy in the efficiency of the generic decision parameter.

The generic decision parameter in this thesis is based on the generalisations made by Rackwitz [38] for $\beta_{t,ULS}$. The question arises as to whether or not the generic decision parameter efficiently decreases the $p_f(d)$ for the ULS. Reliability-based cost optimisation should be performed for the ULS for a range of example structures to determine how well current target values represent optimal design for these structures.

This thesis assumed a lognormal distribution of the action-resistance effect. This choice reflects the assumption that both resistance and load effect have a lower bound at zero. However, the skewness implied by the two parameter lognormal assumption may be too high, particularly for cases where model uncertainty with a normal distribution dominates. Another scenario where the distribution may be different is when the variable load is dominating and has a Gumbel rather than lognormal distribution. It may be useful to assess the influence of distribution type on the assessed optimal $\beta_{t,SLS}$ values.

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A. Cost Optimisation Results

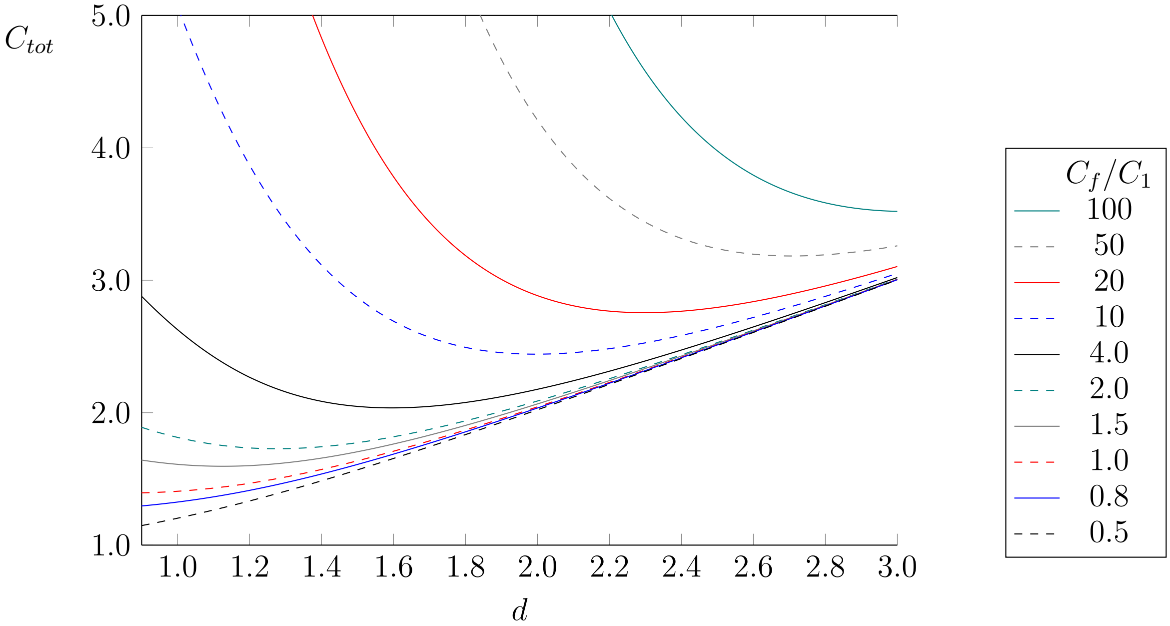

Figure A.1: Cost Optimisation of $V_E = 0.50$

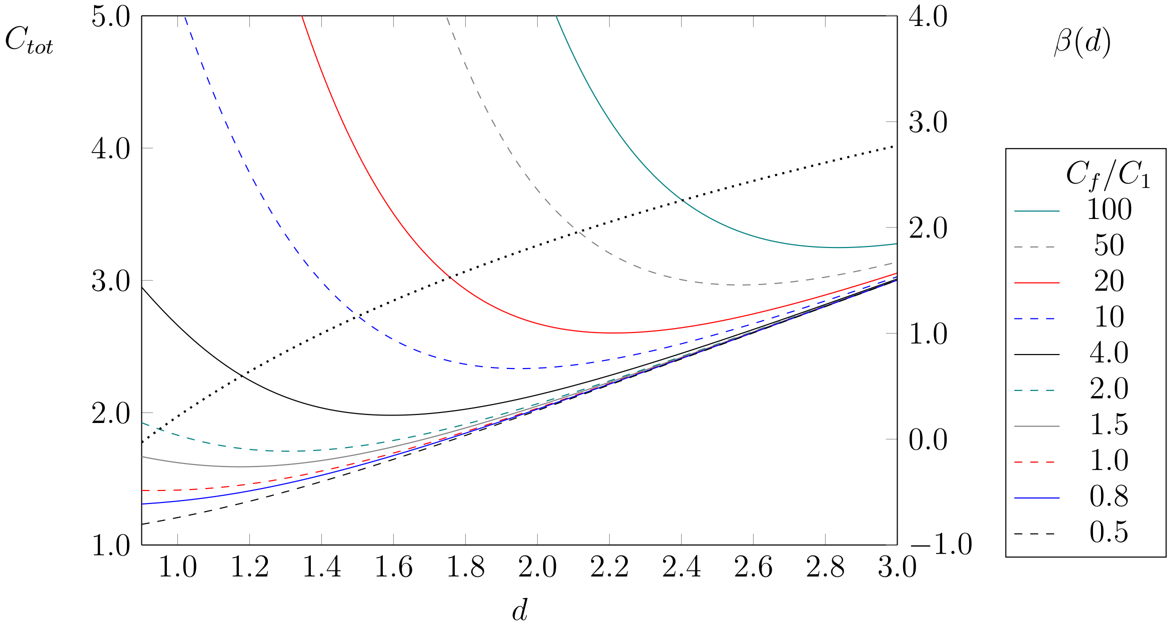

Figure A.2: Cost Optimisation of $V_E = 0.45$

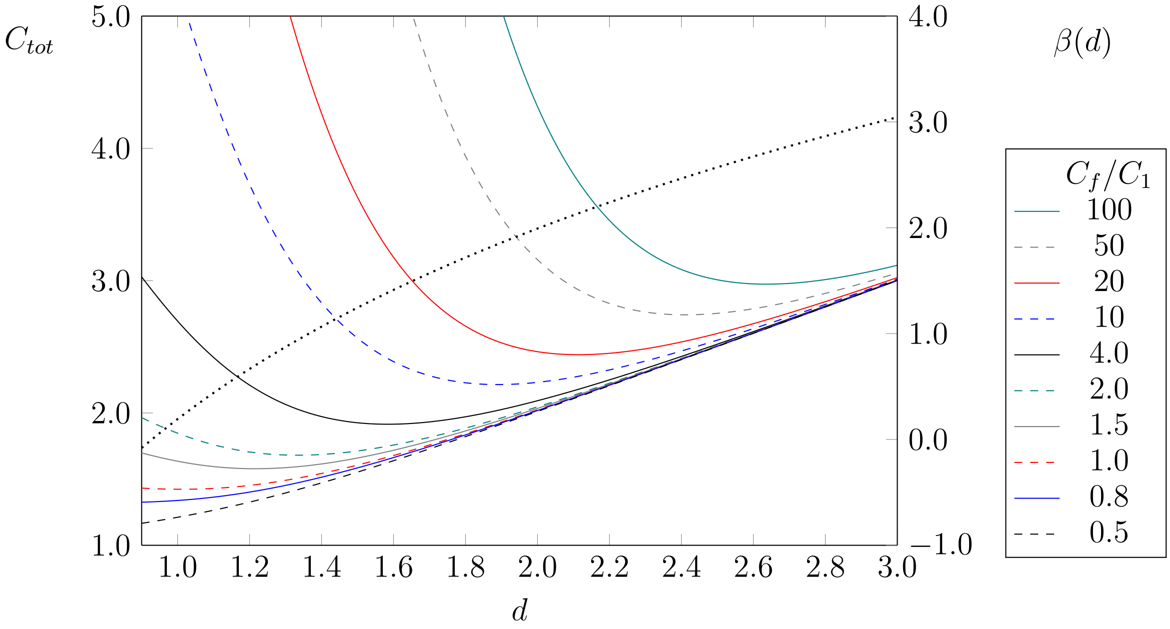

Figure A.3: Cost Optimisation of $V_E = 0.40$

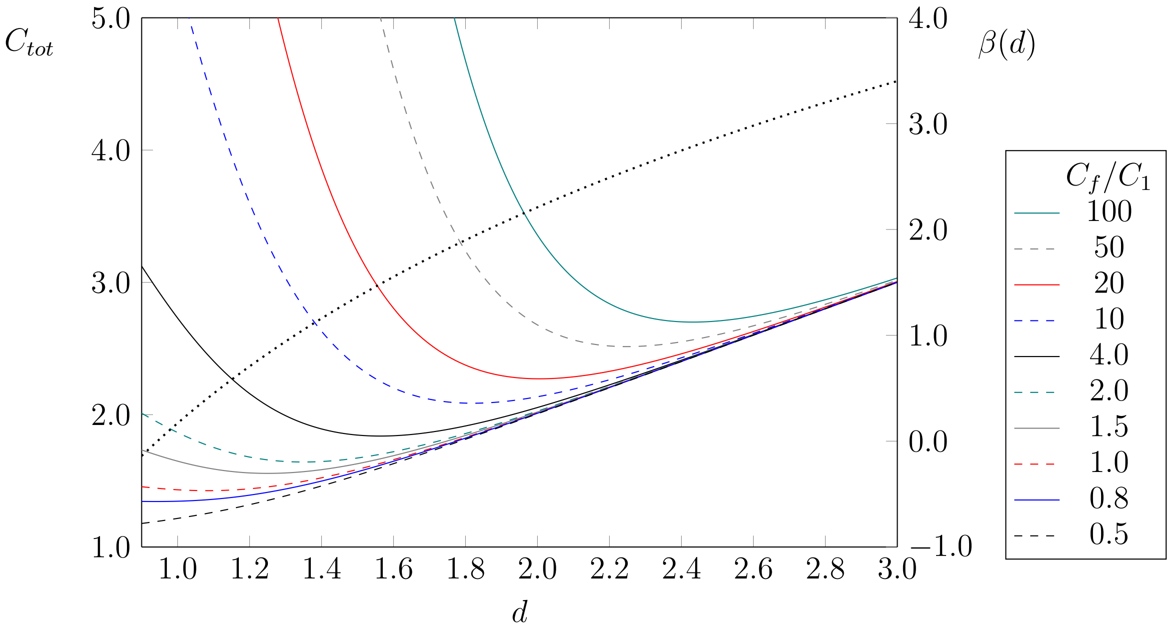

Figure A.4: Cost Optimisation of $V_E = 0.35$

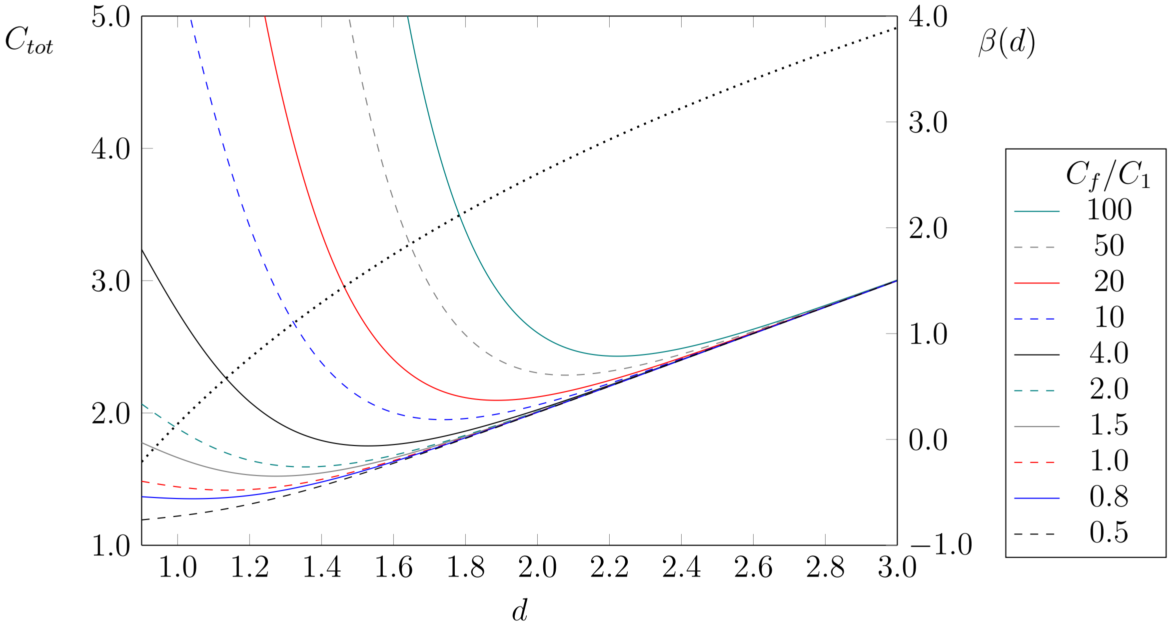

Figure A.5: Cost Optimisation of $V_E = 0.30$

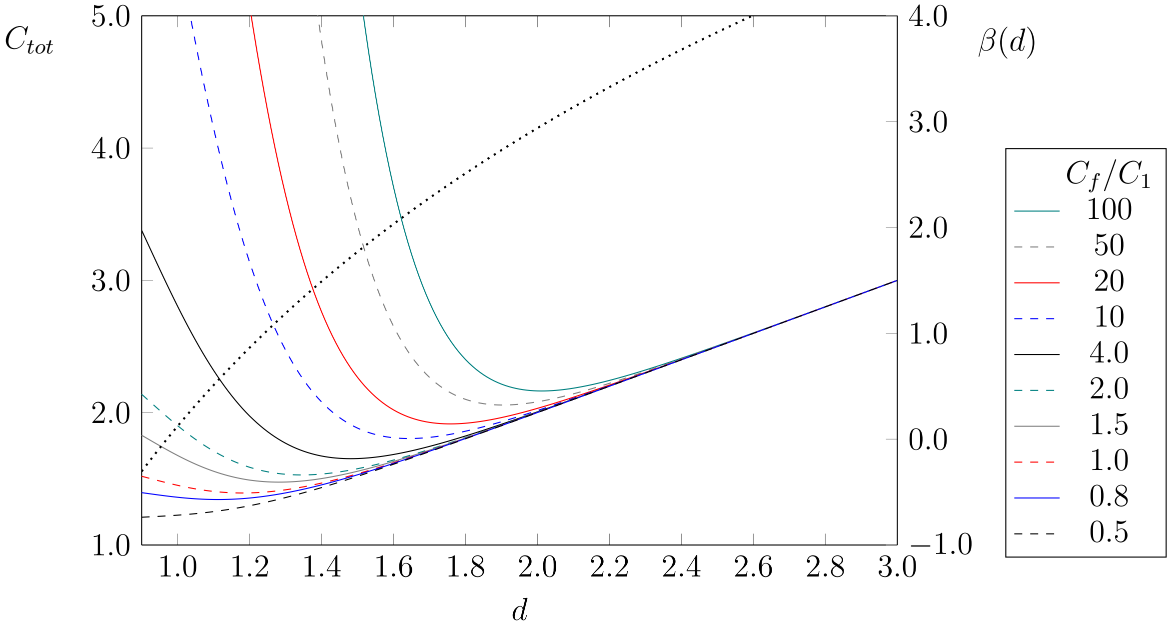

Figure A.6: Cost Optimisation of $V_E = 0.25$

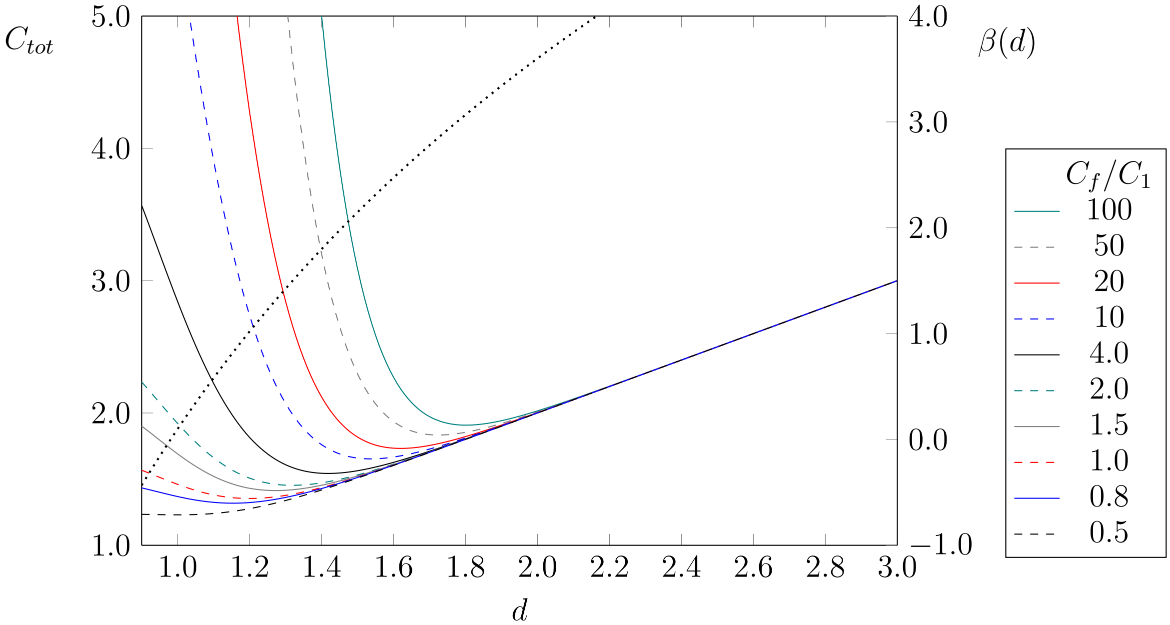

Figure A.7: Cost Optimisation of $V_E = 0.20$

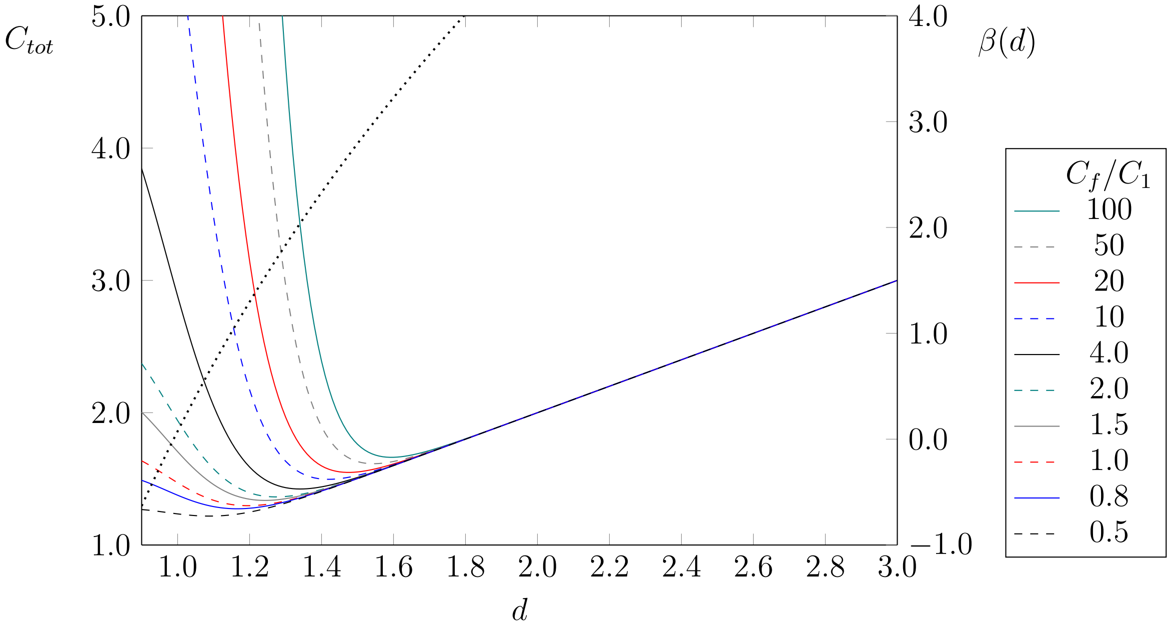

Figure A.8: Cost Optimisation of $V_E = 0.15$

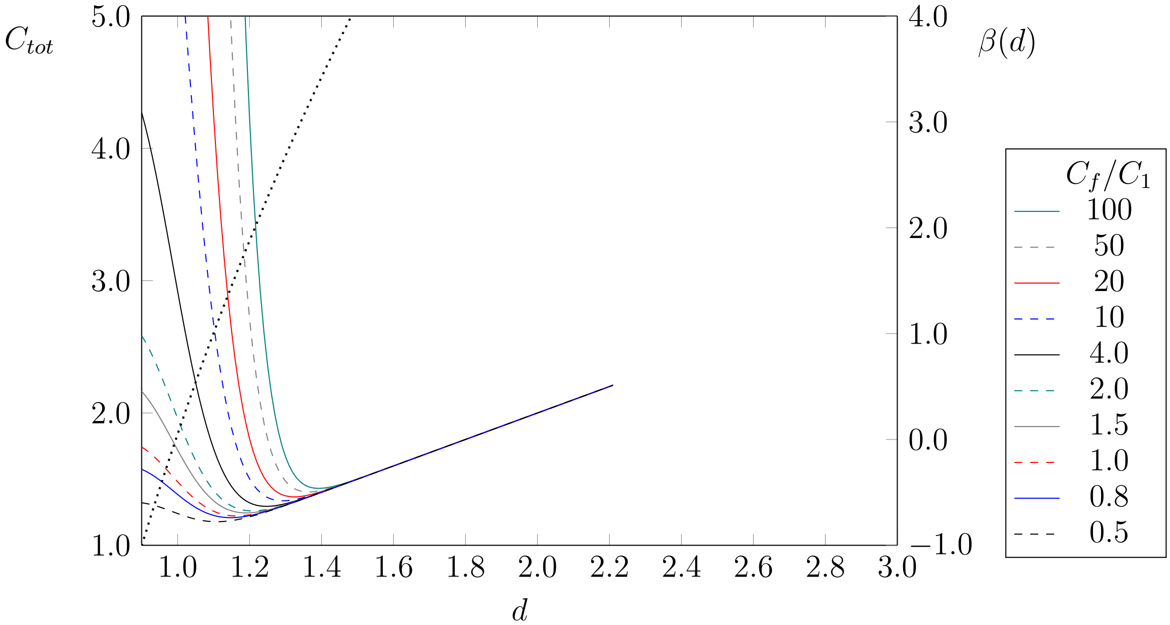

Figure A.9: Cost Optimisation of $V_E = 0.10$

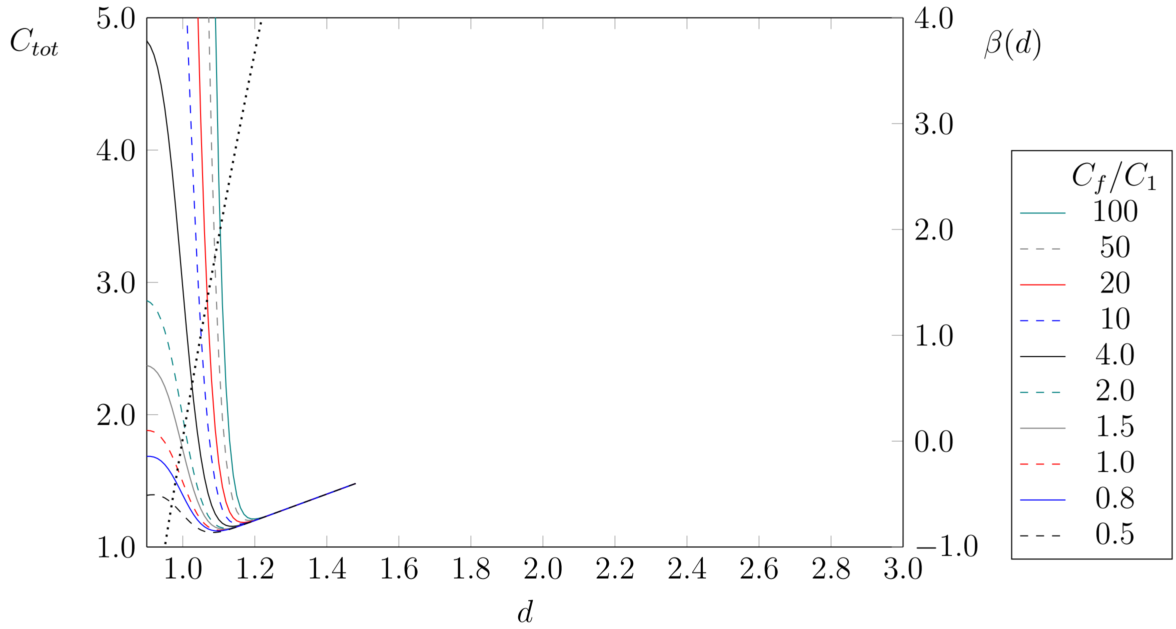

Figure A.10: Cost Optimisation of $V_E = 0.05$

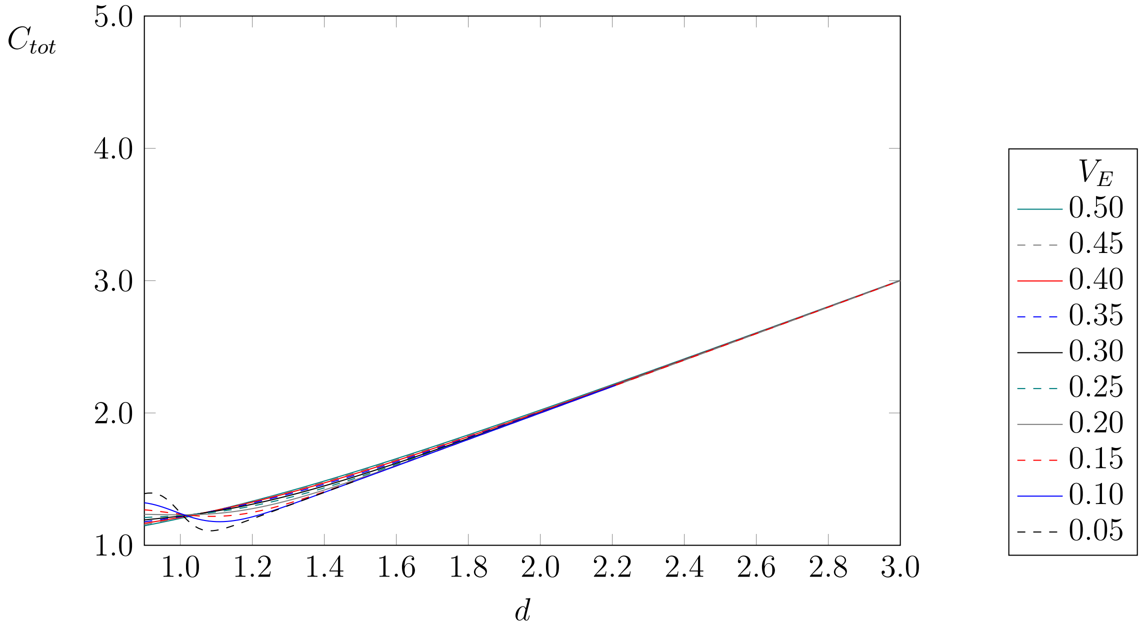

Figure A.11: Cost Optimisation of $C_f/C_1 = 0.5$

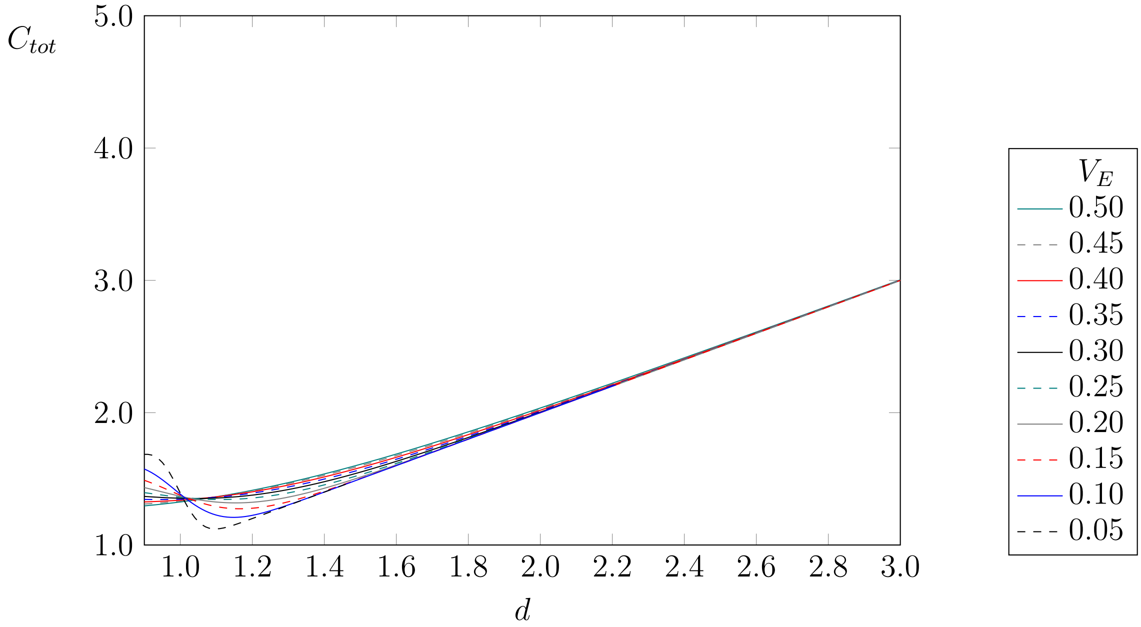

Figure A.12: Cost Optimisation of $C_f/C_1 = 0.8$

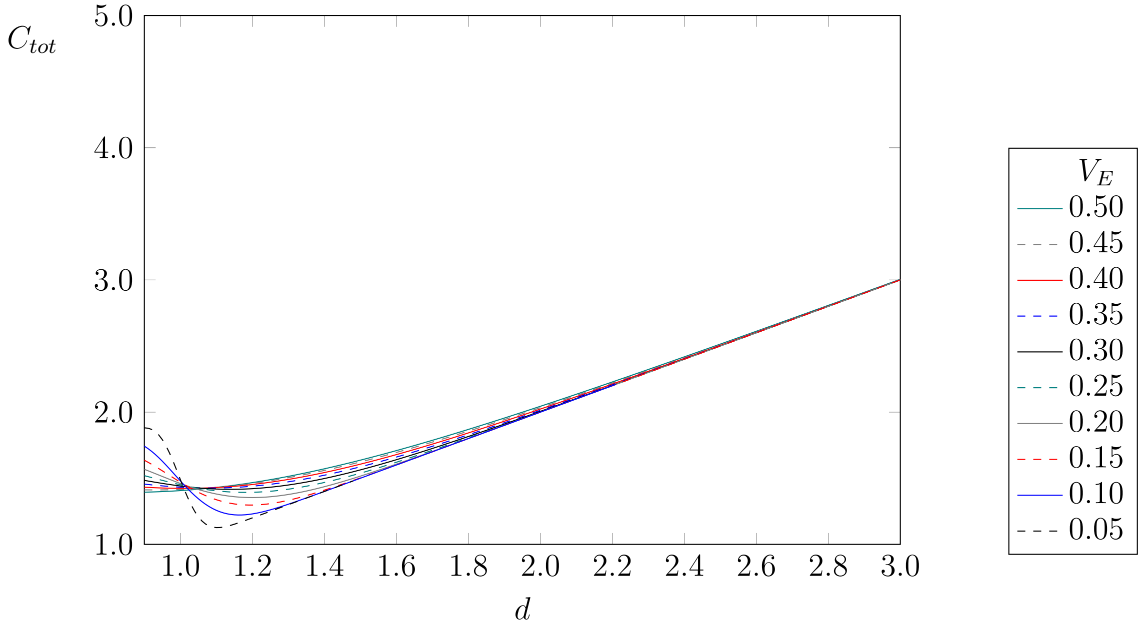

Figure A.13: Cost Optimisation of $C_f/C_1 = 1.0$

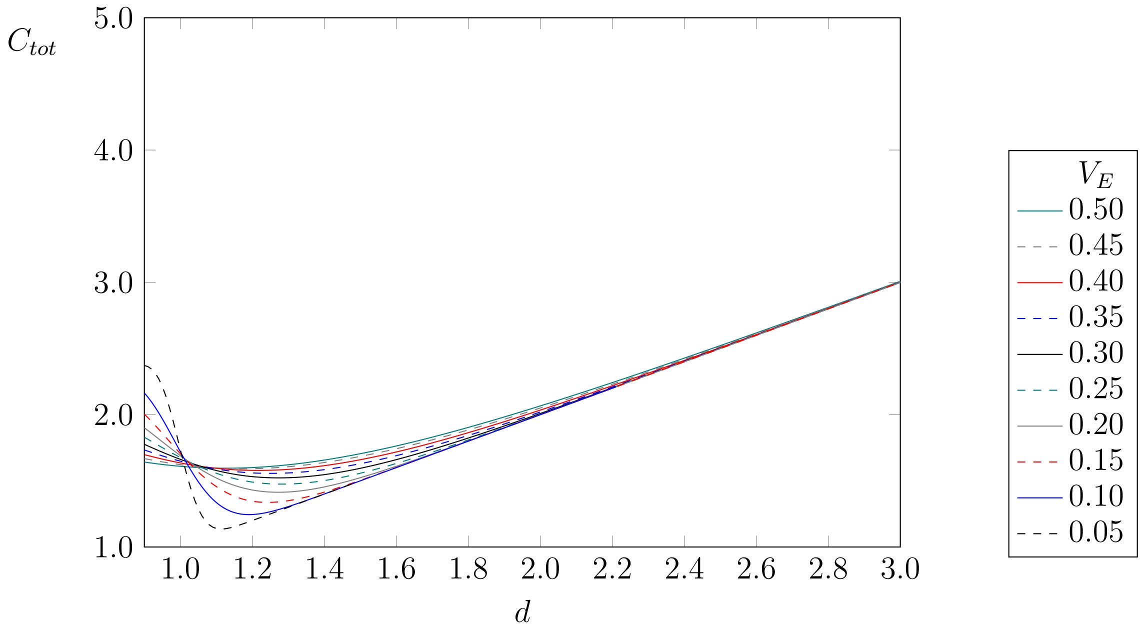

Figure A.14: Cost Optimisation of $C_f/C_1 = 1.5$

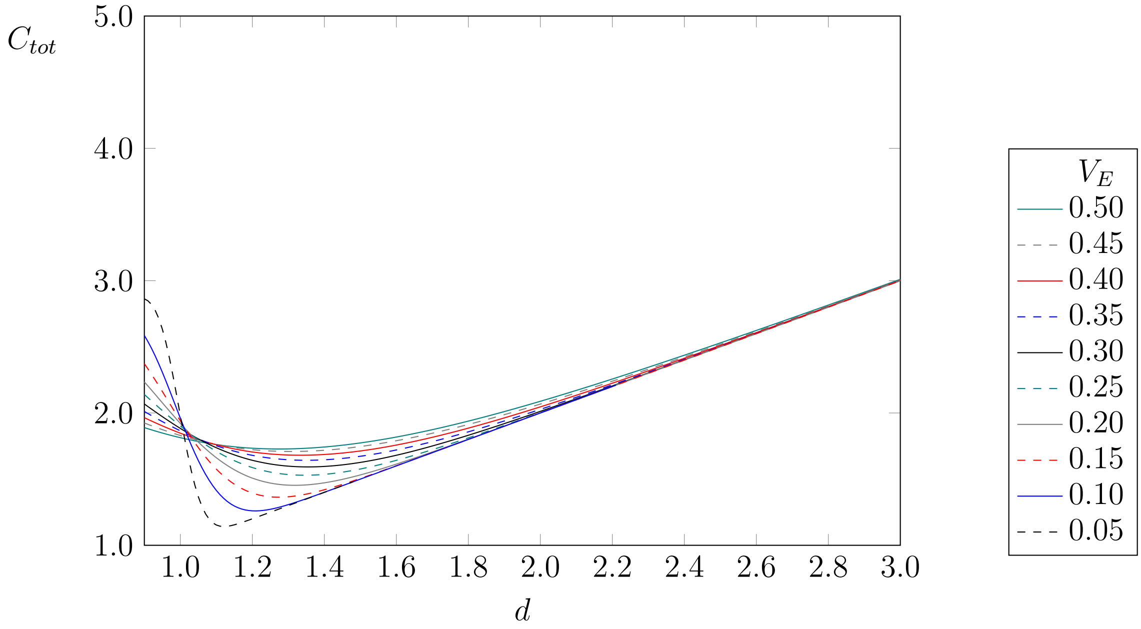

Figure A.15: Cost Optimisation of $C_f/C_1 = 2.0$

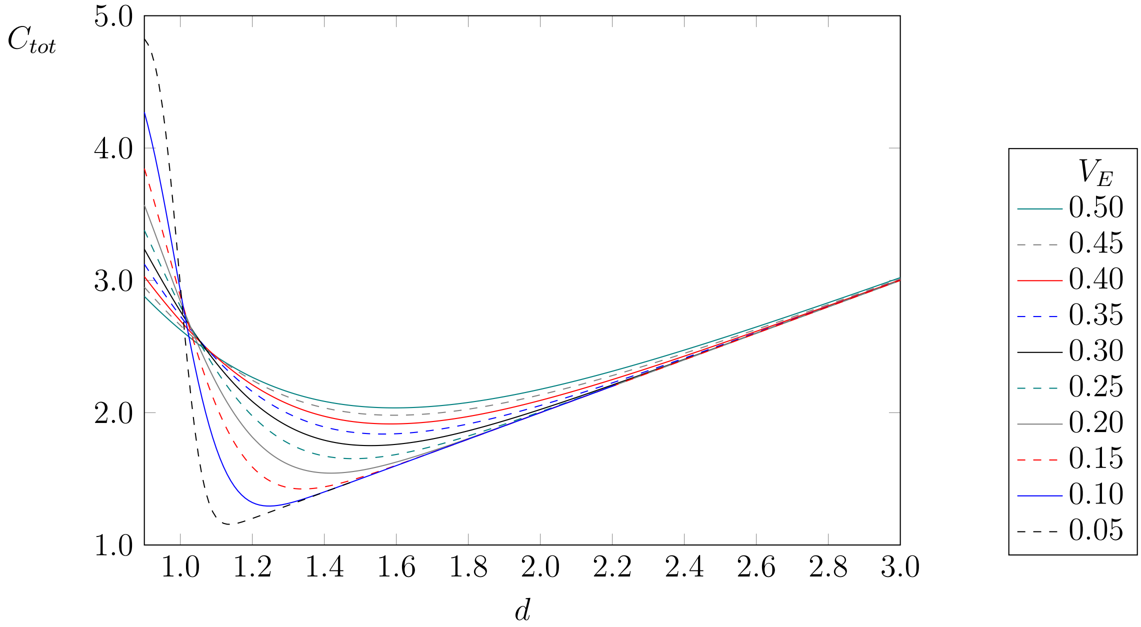

Figure A.16: Cost Optimisation of $C_f/C_1 = 4.0$

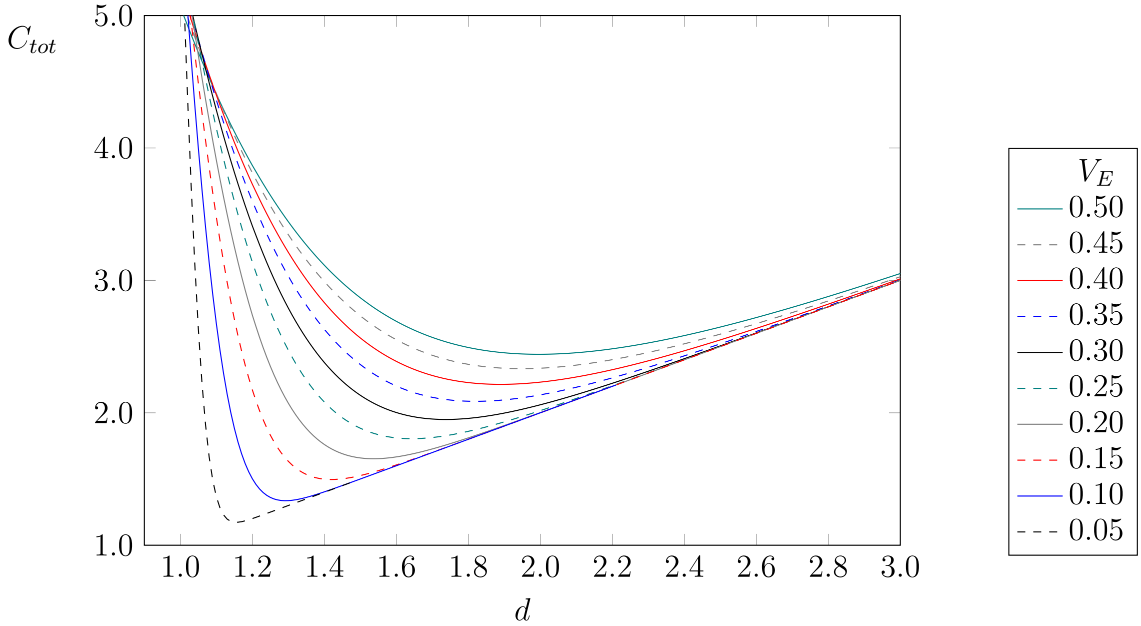

Figure A.17: Cost Optimisation of $C_f/C_1 = 10$

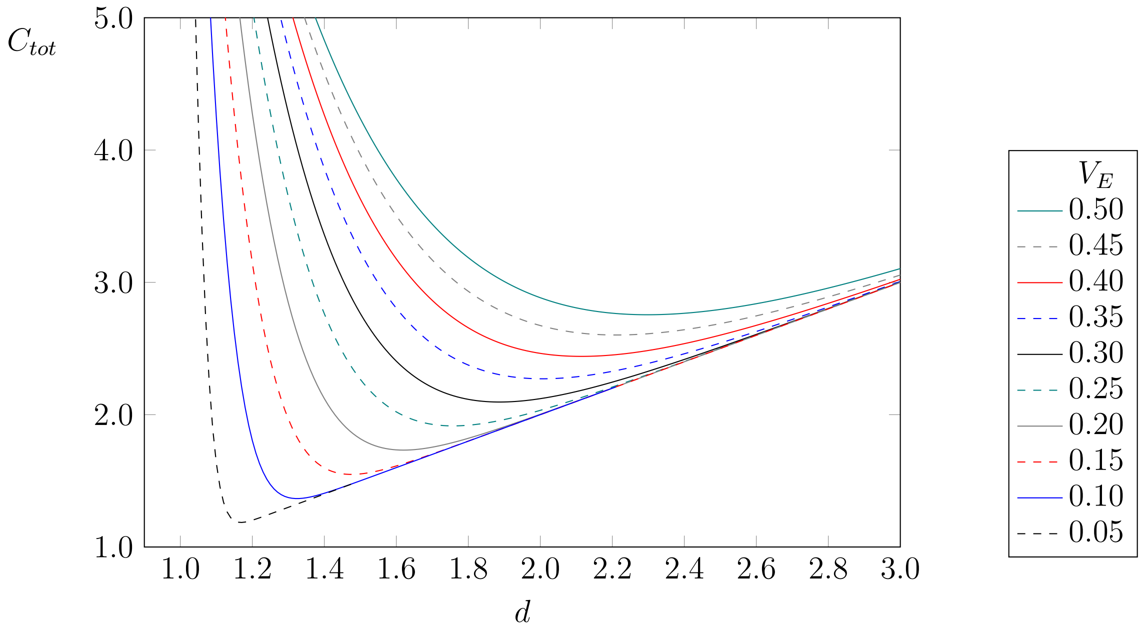

Figure A.18: Cost Optimisation of $C_f/C_1 = 20$

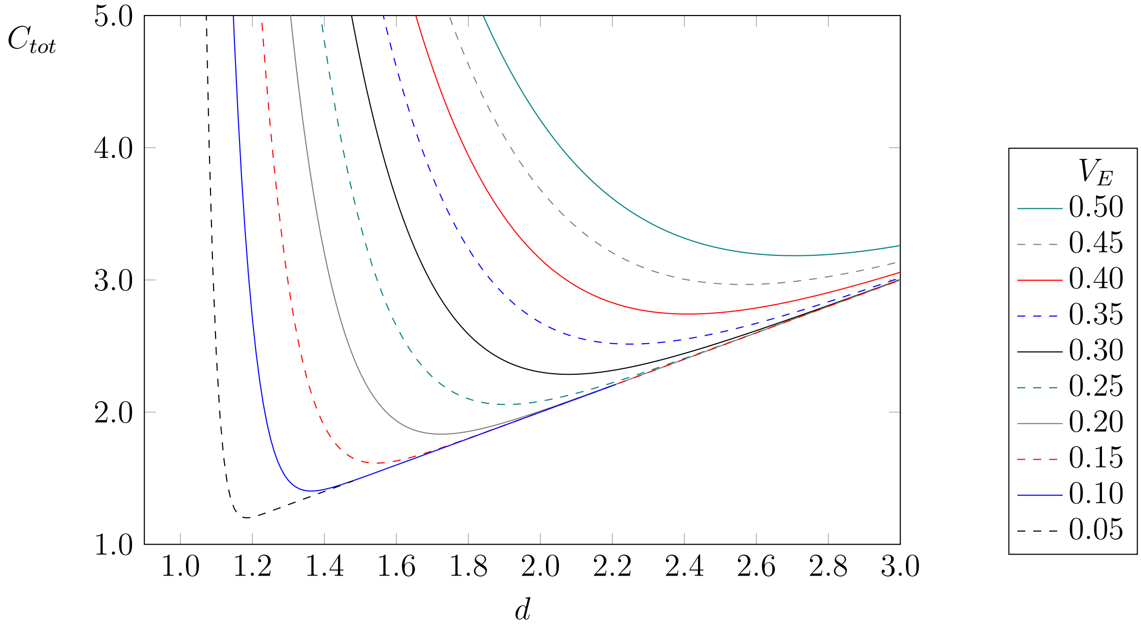

Figure A.19: Cost Optimisation of $C_f/C_1 = 50$

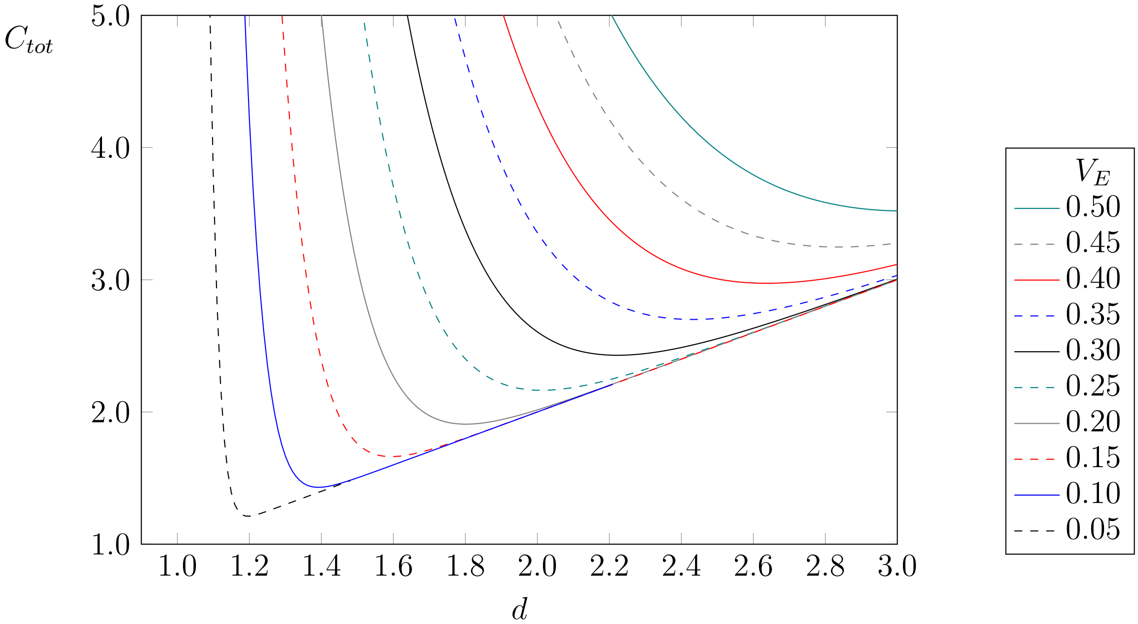

Figure A.20: Cost Optimisation of $C_f/C_1 = 100$

B. WRS Crack Widths

Excel spreadsheets used to calculate $w_{m,max}$.

	WRS 1				WRS 2			
	F-LT	F-ST	T-LT	T-ST	F-LT	F-ST	T-LT	T-ST
h	500	500	500	500	350	350	350	350
phi	20	20	20	20	25	25	25	25
c	40	40	40	40	30	30	30	30
d	450	450	450	450	307.5	307.5	307.5	307.5
z	405	405	405	405	276.75	276.75	276.75	276.75
hc,eff	125	125	125	125	106.25	106.25	106.25	106.25
b	1000	1000	1000	1000	1000	1000	1000	1000
fs	161.30	199.99	165.21	185.25	177.09	207.93	163.57	178.81
Es	200	200	200	200	200	200	200	200
Ecm	31	31	31	31	31	31	31	31
A_e	15.00	6.45	15.00	6.45	15.00	6.45	15.00	6.45
Ap'	0	0	0	0	0	0	0	0
Ac,eff	125000	125000	125000	125000	106250	106250	106250	106250
Pp	0.025	0.021	0.024	0.022	0.040	0.034	0.029	0.026
kt	0.4	0.6	0.4	0.6	0.4	0.6	0.4	0.6
fct	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6
esm	0.000807	0.001000	0.000826	0.000926	0.000885	0.001040	0.000818	0.000894
ecm	0.000282	0.000430	0.000293	0.000412	0.000208	0.000280	0.000259	0.000347
esm - ecm	0.000524	0.000570	0.000533	0.000515	0.000677	0.000760	0.000559	0.000547
k2	0.5	0.5	1	1	0.5	0.5	1	1
sr,max	269.482	301.498	416.856	450.933	208.399	226.925	397.454	424.975
	1.416	1.164	0.9	0.862	1.416	1.164	0.9	0.862
As	**3183.949**	**2568**	**3026.467**	**2698.988**	**4244.041**	**3614.659**	**3056.731**	**2796.27**
E	0.2001	0.2000	0.2001	0.2001	0.1999	0.2008	0.2000	0.2005

Figure B.1: WRS: Calulations to Determine A_{s_L}

Water Retaining Structure 1

Section Properties			Material Properties					
t = h	500.000	mm	fcu	30.000	MPa	phi	20.000	
b = D	20.000	m	fy	450.000	MPa	cover	40.000	
H	5.000	m				As	314.1592654	
			Loading					
V	1570.796	m^3	y_water	10.000	kN/m^2			
			Ms	208.000				

Flexure Long Term

w_lim 0.2 mm

As	2900	3000	3100	3200	3300	3400	3500	3600	3700	3800
h	500	500	500	500	500	500	500	500	500	500
phi	20	20	20	20	20	20	20	20	20	20
c	40	40	40	40	40	40	40	40	40	40
d	450	450	450	450	450	450	450	450	450	450
z	405	405	405	405	405	405	405	405	405	405
hc,eff	125	125	125	125	125	125	125	125	125	125
b	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
fs	177.10	171.19	165.67	160.49	155.63	151.05	146.74	142.66	138.81	135.15
Es	200	200	200	200	200	200	200	200	200	200
Ecm	31	31	31	31	31	31	31	31	31	31
A_e	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Ap'	0	0	0	0	0	0	0	0	0	0
Ac,eff	125000	125000	125000	125000	125000	125000	125000	125000	125000	125000
Pp	0.023	0.024	0.025	0.026	0.026	0.027	0.028	0.029	0.030	0.030
kt	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
fct	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6
esm	0.000885	0.000856	0.000828	0.000802	0.000778	0.000755	0.000734	0.000713	0.000694	0.000676
ecm	0.000302	0.000295	0.000288	0.000281	0.000275	0.000269	0.000264	0.000259	0.000254	0.000249
esm - ecm	0.000583	0.000561	0.000541	0.000521	0.000503	0.000486	0.000470	0.000455	0.000440	0.000427
k2	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
sr,max	282.552	277.667	273.097	268.813	264.788	261.000	257.429	254.056	250.865	247.842
wm	0.165	0.156	0.148	0.140	0.133	0.127	0.121	0.116	0.110	0.106

As	3900	4000	4100	4200	4300	4400	4500	4600	4700	4800
h	500	500	500	500	500	500	500	500	500	500
phi	20	20	20	20	20	20	20	20	20	20
c	40	40	40	40	40	40	40	40	40	40
d	450	450	450	450	450	450	450	450	450	450
z	405	405	405	405	405	405	405	405	405	405
hc,eff	125	125	125	125	125	125	125	125	125	125
b	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
fs	131.69	128.40	125.26	122.28	119.44	116.72	114.13	111.65	109.27	107.00
Es	200	200	200	200	200	200	200	200	200	200
Ecm	31	31	31	31	31	31	31	31	31	31
A_e	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Ap'	0	0	0	0	0	0	0	0	0	0
Ac,eff	125000	125000	125000	125000	125000	125000	125000	125000	125000	125000
Pp	0.031	0.032	0.033	0.034	0.034	0.035	0.036	0.037	0.038	0.038
kt	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
fct	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6
esm	0.000658	0.000642	0.000626	0.000611	0.000597	0.000584	0.000571	0.000558	0.000546	0.000535
ecm	0.000245	0.000241	0.000237	0.000233	0.000229	0.000226	0.000222	0.000219	0.000216	0.000213
esm - ecm	0.000414	0.000401	0.000390	0.000379	0.000368	0.000358	0.000348	0.000339	0.000330	0.000322
k2	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
sr,max	244.974	242.250	239.659	237.190	234.837	232.591	230.444	228.391	226.426	224.542
wm	0.101	0.097	0.093	0.090	0.086	0.083	0.080	0.077	0.075	0.072

Figure B.2: WRS 1: Calculations for F-LT

Water Retaining Structure 2

Section Properties			**Material Properties**					
t = h	350.000	mm	fcu	30.000	MPa	phi	25.000	
b = D	20.000	m	fy	450.000	MPa	cover	30.000	
H	5.000	m				As	490.8738521	
			Loading					
V	1570.796	m^3	y_water	10.000	kN/m^2			
			Ms	208.000				

Flexure Long Term

w_lim 0.2 mm

As	**4100**	**4200**	**4300**	**4400**	**4500**	**4600**	**4700**	**4800**	**4900**	**5000**	**5100**	**5200**	**5300**	**5400**	**5500**	**5600**	**5700**	**5800**	**5900**	**6000**
h	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350
phi	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
c	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30
d	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5
z	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75
hc,eff	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25
b	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
fs	183.31	178.95	174.79	170.81	167.02	163.39	159.91	156.58	153.38	150.32	147.37	144.53	141.81	139.18	136.65	134.21	131.86	129.58	127.39	125.26
Es	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
Ecm	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31
A_e	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Ap'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ac,eff	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250
Pp	0.039	0.040	0.040	0.041	0.042	0.043	0.044	0.045	0.046	0.047	0.048	0.049	0.050	0.051	0.052	0.053	0.054	0.055	0.056	0.056
kt	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
fct	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6
esm	0.000917	0.000895	0.000874	0.000854	0.000835	0.000817	0.000800	0.000783	0.000767	0.000752	0.000737	0.000723	0.000709	0.000696	0.000683	0.000671	0.000659	0.000648	0.000637	0.000626
ecm	0.000213	0.000210	0.000206	0.000204	0.000201	0.000198	0.000196	0.000193	0.000191	0.000189	0.000186	0.000184	0.000182	0.000180	0.000178	0.000177	0.000175	0.000173	0.000172	0.000170
esm - ecm	0.000704	0.000685	0.000667	0.000651	0.000634	0.000619	0.000604	0.000590	0.000576	0.000563	0.000551	0.000538	0.000527	0.000516	0.000505	0.000494	0.000484	0.000475	0.000465	0.000456
k2	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
sr,max	212.137	209.515	207.015	204.628	202.347	200.166	198.077	196.076	194.156	192.313	190.542	188.839	187.200	185.623	184.102	182.636	181.221	179.856	178.536	177.260
wm	0.149	0.144	0.138	0.133	0.128	0.124	0.120	0.116	0.112	0.108	0.105	0.102	0.099	0.096	0.093	0.090	0.088	0.085	0.083	0.081

Figure B.3: WRS 2: Calculations for F-LT

Water Retaining Structure 1

Section Properties

t = h	500.000	mm
b = D	20.000	m
H	5.000	m
V	1570.796	m^3

Material Properties

fcu	30.000	MPa
fy	450.000	MPa

Loading

y_water	10.000	kN/m^2
Ms	500.000	

phi	20.000	
cover	40.000	
As	314.1592654	2513.27

Tension Long Term

w_lim 0.2 mm

As	2900	3000	3100	3200	3300	3400	3500	3600	3700	3800	3900	4000	4100	4200	4300	4400	4500	4600	4700	4800
h	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500	500
phi	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20
c	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40	40
d	450	450	450	450	450	450	450	450	450	450	450	450	450	450	450	450	450	450	450	450
z	405	405	405	405	405	405	405	405	405	405	405	405	405	405	405	405	405	405	405	405
hc,eff	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125	125
b	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
fs	172.41	166.67	161.29	156.25	151.52	147.06	142.86	138.89	135.14	131.58	128.21	125.00	121.95	119.05	116.28	113.64	111.11	108.70	106.38	104.17
Es	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
Ecm	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31
A_e	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Ap'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ac,eff	125000	125000	125000	125000	125000	125000	125000	125000	125000	125000	125000	125000	125000	125000	125000	125000	125000	125000	125000	125000
Pp	0.023	0.024	0.025	0.026	0.026	0.027	0.028	0.029	0.030	0.030	0.031	0.032	0.033	0.034	0.034	0.035	0.036	0.037	0.038	0.038
kt	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
fct	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6
esm	0.000862	0.000833	0.000806	0.000781	0.000758	0.000735	0.000714	0.000694	0.000676	0.000658	0.000641	0.000625	0.000610	0.000595	0.000581	0.000568	0.000556	0.000543	0.000532	0.000521
ecm	0.000302	0.000295	0.000288	0.000281	0.000275	0.000269	0.000264	0.000259	0.000254	0.000249	0.000245	0.000241	0.000237	0.000233	0.000229	0.000226	0.000222	0.000219	0.000216	0.000213
esm - ecm	0.000560	0.000539	0.000519	0.000500	0.000483	0.000466	0.000451	0.000436	0.000422	0.000409	0.000396	0.000385	0.000373	0.000362	0.000352	0.000342	0.000333	0.000324	0.000316	0.000307
k2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
sr,max	429.103	419.333	410.194	401.625	393.576	386.000	378.857	372.111	365.730	359.684	353.949	348.500	343.317	338.381	333.674	329.182	324.889	320.783	316.851	313.083
wm	0.240	0.226	0.213	0.201	0.190	0.180	0.171	0.162	0.154	0.147	0.140	0.134	0.128	0.123	0.118	0.113	0.108	0.104	0.100	0.096

Figure B.4: WRS 1: Calculations for T-LT

Water Retaining Structure

Section Properties			Material Properties					
t = h	350.000	mm	fcu	30.000	MPa	phi	25.000	
b = D	20.000	m	fy	450.000	MPa	cover	30.000	
H	5.000	m				As	490.8738521	
			Loading					
V	1570.796	m^3	y_water	10.000	kN/m^2			
			Ms	500.000				

Tension Long Term

w_lim 0.2 mm

As	3000	3100	3200	3300	3400	3500	3600	3700	3800	3900	4000	4100	4200	4300	4400	4500	4600	4700	4800	4900
h	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350
phi	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25
c	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30
d	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5	307.5
z	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75	276.75
hc,eff	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25	106.25
b	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000
fs	166.67	161.29	156.25	151.52	147.06	142.86	138.89	135.14	131.58	128.21	125.00	121.95	119.05	116.28	113.64	111.11	108.70	106.38	104.17	102.04
Es	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
Ecm	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31
A_e	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Ap'	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Ac,eff	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250	106250
Pp	0.028	0.029	0.030	0.031	0.032	0.033	0.034	0.035	0.036	0.037	0.038	0.039	0.040	0.040	0.041	0.042	0.043	0.044	0.045	0.046
kt	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
fct	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6	2.6
esm	0.000833	0.000806	0.000781	0.000758	0.000735	0.000714	0.000694	0.000676	0.000658	0.000641	0.000625	0.000610	0.000595	0.000581	0.000568	0.000556	0.000543	0.000532	0.000521	0.000510
ecm	0.000262	0.000256	0.000251	0.000245	0.000241	0.000236	0.000231	0.000227	0.000223	0.000220	0.000216	0.000213	0.000210	0.000206	0.000204	0.000201	0.000198	0.000196	0.000193	0.000191
esm - ecm	0.000571	0.000550	0.000531	0.000512	0.000495	0.000478	0.000463	0.000448	0.000435	0.000421	0.000409	0.000397	0.000386	0.000375	0.000365	0.000355	0.000345	0.000336	0.000328	0.000319
k2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
sr,max	403.042	393.331	384.227	375.674	367.625	360.036	352.868	346.088	339.664	333.571	327.781	322.274	317.030	312.029	307.256	302.694	298.332	294.154	290.151	286.311
wm	0.230	0.216	0.204	0.192	0.182	0.172	0.163	0.155	0.148	0.141	0.134	0.128	0.122	0.117	0.112	0.107	0.103	0.099	0.095	0.091

Figure B.5: WRS 2: Calculations for T-LT

C. SSB Deflections

Excel spreadsheets used to calculate δ_{max}).

L	m	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5	7.5
b	mm	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350	350
h	**mm**	**470**	**475**	**480**	**485**	**490**	**495**	**500**	**505**	**510**	**515**	**520**	**525**	**530**	**535**	**540**	**545**	**550**
Asprov	mm^2	2412.743	2412.743	2412.743	2412.743	2412.743	2412.743	2412.743	2412.743	2412.743	2412.743	2412.743	2412.743	2412.743	2412.743	2412.743	2412.743	2412.743
LL	kN/m	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
DL	kN/m	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
d	mm	422.5	427.5	432.5	437.5	442.5	447.5	452.5	457.5	462.5	467.5	472.5	477.5	482.5	487.5	492.5	497.5	502.5
p	-	0.016316	0.016125	0.015939	0.015757	0.015579	0.015405	0.015234	0.015068	0.014905	0.014746	0.01459	0.014437	0.014287	0.014141	0.013997	0.013856	0.013719
a_e	-	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
x	mm	209.75	211.40	213.04	214.67	216.29	217.90	219.51	221.11	222.70	224.28	225.85	227.42	228.98	230.53	232.07	233.61	235.14
B	-	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
fcu	Mpa	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38	38
Mcr	kNm	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01	4.01
M	kNm	51.63	52.74	53.85	54.98	56.12	57.27	58.43	59.61	60.79	61.99	63.20	64.42	65.66	66.90	68.16	69.42	70.70
		91.41	91.41	91.41	91.41	91.41	91.41	91.41	91.41	91.41	91.41	91.41	91.41	91.41	91.41	91.41	91.41	91.41
e	-	0.84	0.83	0.83	0.82	0.81	0.80	0.80	0.79	0.78	0.77	0.76	0.75	0.74	0.73	0.72	0.71	0.70
q	kN/m = N/	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13
L	mm	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500	7500
Eceff	MPa	13000	13000	13000	13000	13000	13000	13000	13000	13000	13000	13000	13000	13000	13000	13000	13000	13000
I1	mm^4	3.03E+09	3.13E+09	3.23E+09	3.33E+09	3.43E+09	3.54E+09	3.65E+09	3.76E+09	3.87E+09	3.98E+09	4.1E+09	4.22E+09	4.34E+09	4.47E+09	4.59E+09	4.72E+09	4.85E+09
I2	mm^4	1.19E+09	1.21E+09	1.24E+09	1.27E+09	1.3E+09	1.33E+09	1.36E+09	1.4E+09	1.43E+09	1.46E+09	1.49E+09	1.52E+09	1.56E+09	1.59E+09	1.62E+09	1.66E+09	1.69E+09
0.37 I1	mm^4	1.12E+09	1.16E+09	1.19E+09	1.23E+09	1.27E+09	1.31E+09	1.35E+09	1.39E+09	1.43E+09	1.47E+09	1.52E+09	1.56E+09	1.61E+09	1.65E+09	1.7E+09	1.75E+09	1.8E+09
Actual % I1		0.39	0.39	0.39	0.38	0.38	0.38	0.37	0.37	0.37	0.37	0.36	0.36	0.36	0.36	0.35	0.35	0.35

Figure C.1: Calculations for Deflections

The derivation of the deflection equation inputted into RT is shown below:

$$\delta_{max} = \xi \frac{5}{384} \frac{qL^4}{E_{c,eff} I_2} + (1-\xi) \frac{5}{384} \frac{qL^4}{E_{c,eff} I_1} \tag{C.1}$$

with:

$$I_1 = \frac{bh^3}{12} \tag{C.2}$$

$$I_2 = f I_1 \tag{C.3}$$

Substituting and rewriting:

$$\begin{aligned} \delta_{max} =& \xi \frac{5}{384} \frac{qL^4}{E_{c,eff}(\frac{fbh^3}{12})} + (1-\xi) \frac{5}{384} \frac{qL^4}{E_{c,eff}(\frac{bh^3}{12})} \\ =& \frac{5}{384} \frac{12qL^4}{E_{c,eff}(bh^3)} [1 + \xi(\frac{1}{f} - 1)] \\ =& \frac{5}{32} \frac{qL^4}{E_{c,eff} bh^3} [1 + \xi(\frac{1}{f} - 1)] \end{aligned} \tag{C.4}$$

Determining ξ:

$$M_{cr} = \frac{0.65\sqrt{f_{cu}} I_1}{h/2} \tag{C.5}$$

$$M = \frac{qL^2}{8} \tag{C.6}$$

$$\begin{aligned} \xi &= 1 - \beta_\xi (\frac{M_{cr}}{M_s})^2 \\ &= 1 - \beta_\xi (\frac{\frac{0.65\sqrt{f_{cu}} \frac{bh^3}{12}}{h/2}}{\frac{qL^2}{8}})^2 \\ &= 1 - \frac{169}{225} \beta_\xi (\frac{\sqrt{f_{cu}} bh^2}{wL^2})^2 \end{aligned} \tag{C.7}$$

Substituting into the deflection equation and subbing in $q = DL + LL$ results in:

$$\delta_{max,RT} = \frac{5(G+Q)L^4}{32 E_{c,eff} bh^3} [1 + (\frac{1}{f} - 1)(1 - \frac{169 \beta_\xi f_{cu}}{225} (\frac{bh^2}{(G+Q)L^2})^2)] \tag{C.8}$$